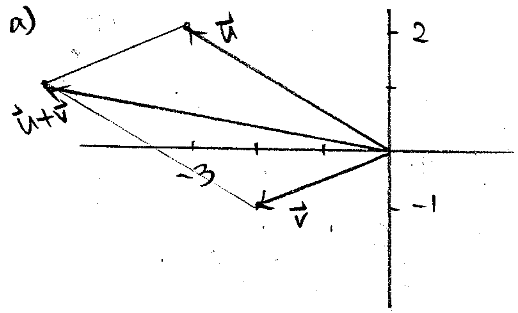
P 6.7

6. $\vec{u} = (-3, 2)$, $\vec{v} = (-2, -1)$

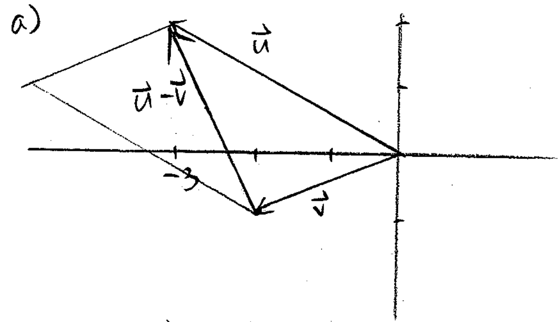
a)



b) $\vec{u} + \vec{v} = (-3-2, 2+(-1))$
$= (-5, 1)$

8.

a)



b) $\vec{u} - \vec{v} = (-3-(-2), 2-(-1))$
$= (-1, 3)$

12. $\vec{u} = (3, -5)$, $\vec{v} = (-4, 3)$

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}|\cdot|\vec{v}|} = \frac{-12-15}{\sqrt{3^2+(-5)^2}\sqrt{(-4)^2+3^2}}$$

$$= \frac{-27}{\sqrt{34}\cdot 5}, \quad \theta = \cos^{-1}\left(\frac{-27}{\sqrt{34}\cdot 5}\right).$$

14. $|7+12i| = \sqrt{7^2+12^2}$

$= \sqrt{49+144}$

$= \sqrt{193}$

17. $(2-2i)^{333}$

$r = |2-2i| = \sqrt{2^2+(-2)^2}$

$= \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$2-2i = r\cos\theta + i r\sin\theta$

$\cos\theta = \frac{2}{r} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\sin\theta = \frac{-2}{r} = \frac{-2}{2\sqrt{2}} = -\frac{1}{\sqrt{2}}$

$\theta = -\frac{\pi}{4}.$

$2-2i = 2\sqrt{2}\cos(-\frac{\pi}{4}) + 2\sqrt{2}\sin(-\frac{\pi}{4})i$

$(2-2i)^{333} = (2\sqrt{2})^{333}\left(\cos\left(-\frac{333\pi}{4}\right) + i\sin\left(-\frac{333\pi}{4}\right)\right)$

$-\frac{333}{4} = -83\frac{1}{4} = -84 + \frac{3}{4}$. since even multiples of π can be discarded when computing values of cos, sin

$(2-2i)^{333} = (2\sqrt{2})^{333}\left(\cos(\frac{3}{4}\pi) + i\sin(\frac{3}{4}\pi)\right)$

$= (2\sqrt{2})^{333}\left(-\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)$

$= 2^{499}(-1+i)$.

19. $z^4 = -2$, let $z = r(\cos\theta + i\sin\theta)$

$r^4(\cos 4\theta + i\sin 4\theta) = -2$

$r^4 = 2$, $\cos 4\theta = -1$

$r = 2^{\frac{1}{4}}$, $4\theta = \pi + 2k\pi$, k is integer

$\theta = \frac{\pi}{4} + \frac{k\pi}{2}$

So $\theta = \frac{\pi}{4}, \frac{3}{4}\pi, \frac{5}{4}\pi, \frac{7}{4}\pi$

$z = 2^{\frac{1}{4}}(\cos\theta + i\sin\theta)$

$z = 2^{\frac{1}{4}}\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right),\ 2^{\frac{1}{4}}\left(-\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right),\ 2^{\frac{1}{4}}\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right);\ 2^{\frac{1}{4}}\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right).$

20. $z^3 = 4i$, $z = r(\cos\theta + i\sin\theta)$

$r^3(\cos 3\theta + i\sin 3\theta) = 4i$

$r^3 = 4$, $\cos 3\theta = 0$, $\sin 3\theta = 1$

$r = 4^{\frac{1}{3}}$, $3\theta = \frac{\pi}{2} + 2k\pi$, k is integer

$\theta = \frac{\pi}{6} + \frac{2k\pi}{3}$ So $\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

$z = 4^{\frac{1}{3}}(\cos\theta + i\sin\theta)$

$z = 4^{\frac{1}{3}}\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right),\ 4^{\frac{1}{3}}\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right),\ -4^{\frac{1}{3}}i.$

44. $z^3 = c > 0$, c real.

$r^3(\cos 3\theta + i\sin 3\theta) = c > 0$

$r^3 = c$, $\cos 3\theta = 1$, $\sin 3\theta = 0$

$r^3 = c$, $3\theta = 2k\pi$, k is integer

$\theta = \frac{2k\pi}{3}$. So $\theta = 0, \frac{2\pi}{3}$, or $\frac{4\pi}{3}$

$z = c^{\frac{1}{3}}(\cos\theta + i\sin\theta)$

$z = c^{\frac{1}{3}},\ c^{\frac{1}{3}}\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right),\ c^{\frac{1}{3}}\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$

subset $= \left\{ d,\ d\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right),\ d\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) \mid d \text{ is real positive} \right\}$