Homework 2

4.3

4. $f'(x) = (x-1)^2(x+2)^2$

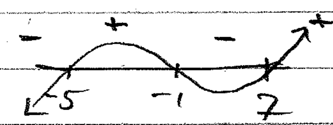
a) CP : $1, -2$

b) Intervals of increase / decrease :

	$(-\infty, -2)$	$(-2, 1)$	$(1, \infty)$
f'	+	+	+
f	inc	inc	inc

c) There are no local extrema

6. $f'(x) = (x-7)(x+1)(x+5) \longrightarrow$



a) CP : $-5, -1, 7$

b)

	$(-\infty, -5)$	$(-5, -1)$	$(-1, 7)$	$(7, \infty)$
f'	−	+	−	+
f	dec	inc	dec	inc

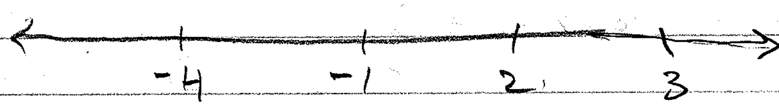
c) -5 and 7 are local min

-1 local max

8. $f'(x) = \dfrac{(x-2)(x+4)}{(x+1)(x-3)}$ $\quad x \neq -1, 3$

a) CP : $2, -4$

b)



	$(-\infty, -4)$	$(-4, -1)$	$(-1, 2)$	$(2, 3)$	$(3, \infty)$
f'	+	−	+	−	+
f	inc	dec	inc	dec	inc

reasoning:

i) if $x < -4$, then $x-2 < 0$, $x+1 < 0$, $x-2 < 0$ and $x-3 < 0$

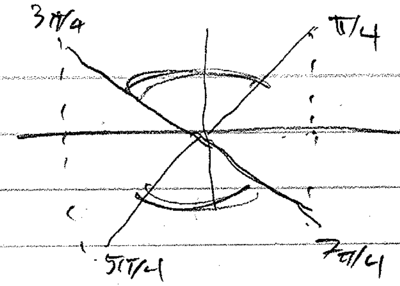
ii) if $-4 < x < -1$, then $(x+4) > 0$ but $x+1 < 0$,
$x-3 < 0$, $x-2 < 0$

iii) etc. fill in details

c) $-4, 2$ are local max

assuming f is defined at $-1, 3$, then these are local min.

$-\frac{1}{\sqrt{2}} < \cos x < \frac{1}{\sqrt{2}}$



when $\pi/4 < x < 3\pi/4$

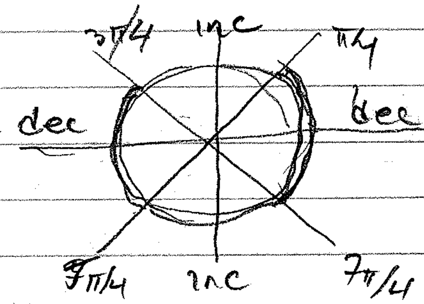
or

$5\pi/4 < x < 7\pi/4$

Similarly $f'(x) < 0$ when $x \in (0, \pi/4) \cup (3\pi/4, 5\pi/4) \cup (7\pi/4, 2\pi)$

$0 \quad \pi/4 \quad 3\pi/4 \quad 5\pi/4 \quad 7\pi/4 \quad 2\pi$

(−)	(+)	(−)	(+)	(−)
dec	inc	dec	inc	dec



So local max occur at

$x = 3\pi/4,\ 7\pi/4,\ 0$

local min at

$x = \pi/4,\ 5\pi/4,\ 2\pi$

20. $g(t) = -3t^2 + 9t + 5$

$g'(t) = -6t + 9$. $g'(t) = 0$ when $t = \frac{9}{6} = \frac{3}{2}$

C.P. $x = \frac{3}{2}$

	$(-\infty, \frac{3}{2})$	$(\frac{3}{2}, \infty)$
f'	+	−
f	inc	dec

so $3/2$ is a local max

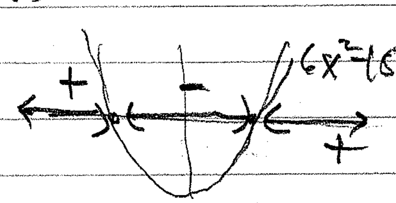
22. $h(x) = 2x^3 - 18x$

$h'(x) = 6x^2 - 18 = 0$ when $x^2 = 3$

$|x| = \sqrt{3}$

The C.P. are $x_1 = \sqrt{3}$, $x_2 = -\sqrt{3}$

	$(-\infty, -\sqrt{3})$	$(-\sqrt{3}, \sqrt{3})$	$(\sqrt{3}, \infty)$
f'	+	−	+
f	inc	dec	inc



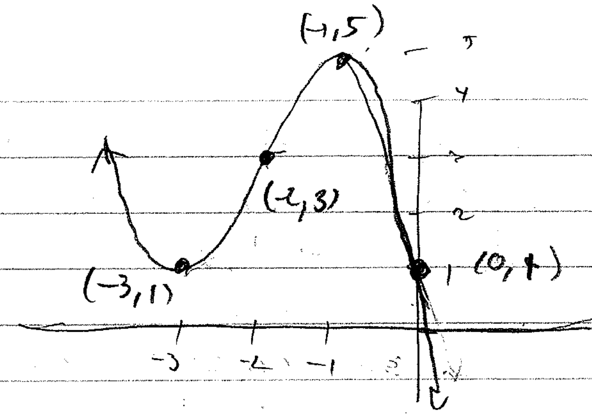
6. No asymptotes

7. y-intercept : $(0, 1)$

$(-2, y(-2)) = \quad ,$

$(-2, 1 + (8 - 24 + 8))$

$(-2, 3)$



15. $y = (x-2)^3 + 1$ domain $= \mathbb{R}$

- $y' = 3(x-2)^2$

  CP: $x = 2$ → 1st Der. test:

	$(-\infty, 2)$	$(2, \infty)$
f'	+	+
f	inc	inc

2 is neither a min or max.

- $y'' = 6(x-2)$

  $x = 2$ is point of inflection

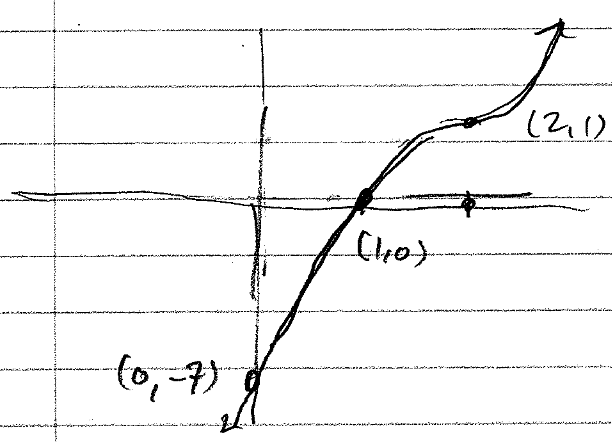
	$(-\infty, 2)$	$(2, \infty)$
concavity:	down	up

- There are no asymptotes
- Key points:

  y intercept: $(0, -7)$

  x-intercepts : $(x-2)^3 = -1 \quad x - 2 = -1 \quad x = 1$

  $(1, 0)$



40. $y = x^2 + \frac{2}{x}$ Domain $= \mathbb{R} - \{0\}$

- $y' = 2x - \frac{2}{x^2}$ Domain $= \mathbb{R} - \{0\}$

$y'' = 2 + \frac{4}{x^3}$ Domain: $\mathbb{R} - \{0\}$

- Critical points: $y' = 0 \Rightarrow 2x = \frac{2}{x^2} \Leftrightarrow x^3 = 1 \Leftrightarrow x = 1$

	$(-\infty, 0)$	$(0, 1)$	$(1, \infty)$
f'	$-$	$-$	$+$
f	dec	dec	inc

so 1 is a local min

- Inflection point occurs when $x^3 = -2$

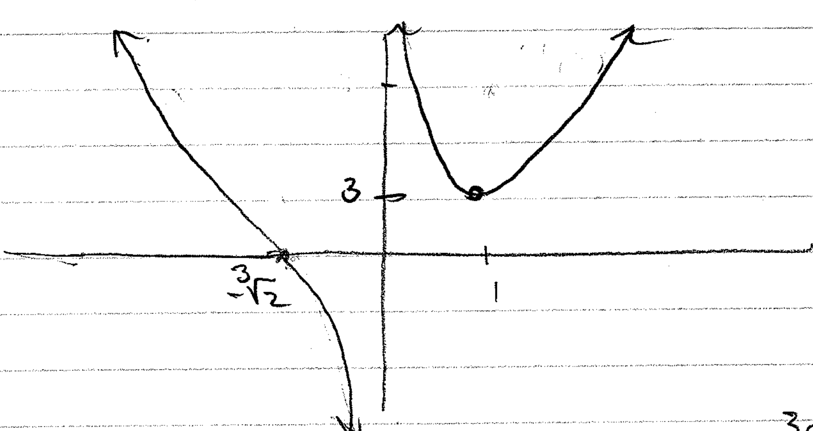
I.P.: $x = -\sqrt[3]{2}$

	$(-\infty, -\sqrt[3]{2})$	$(-\sqrt[3]{2}, 0)$	$(0, \infty)$
f''	$+$	$-$	$+$
concavity	up	down	up

- Asymptotes:

$\rightarrow$ Vertical asymptotes: $x = 0$

$\lim_{x \to 0^+} x^2 + \frac{2}{x} = +\infty$ $\lim_{x \to 0^-} x^2 + \frac{2}{x} = -\infty$



Y-intercept (none) X-intercept $x = -\sqrt[3]{2}$

- Asymptotes:

→ vertical: $x = 2$ $\quad \lim_{x \to 2^+} \frac{x^2-3}{x-2} = +\infty$

$$\lim_{x \to 2^-} \frac{x^2-3}{x-2} = -\infty$$

→ slant:

$$\begin{array}{r} x+2 \\ x-2 \;) \; x^2-3 \\ x^2-2x \\ 2x-3 \\ 2x-4 \\ 1 \end{array}$$

$\Rightarrow (x^2-3) = (x-2)(x+2) + 1$

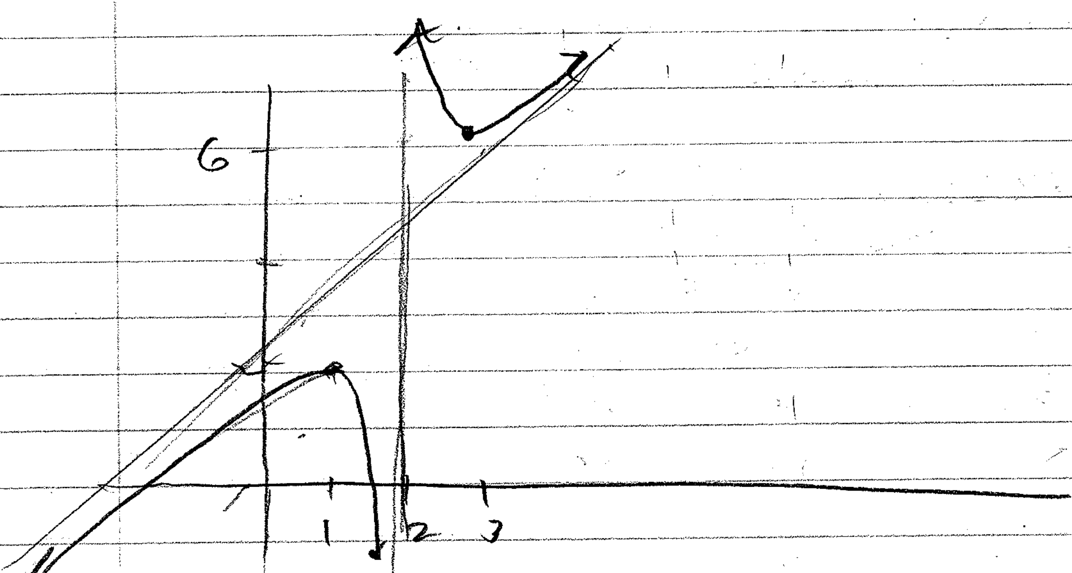
So $\frac{x^2-3}{x-2} = (x+2) + \frac{1}{x-2}$ $\quad x \neq 2$

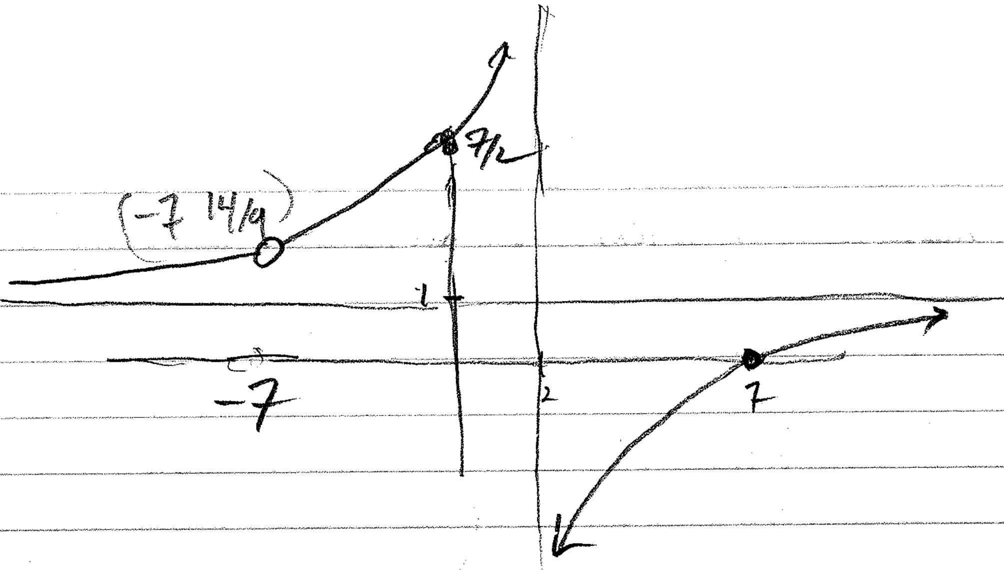
So for $|x|$ large, $\frac{x^2-3}{x-2} \approx x+2$

If $x \gg 2$ $f(x)$ approaches $x+2$ from above

If $x \ll 2$ $f(x)$ approaches $x+2$ from below

because $\frac{1}{x-2} < 0$





80 $f(x) = \dfrac{x^2}{x^2-1} = \dfrac{x^2}{(x+1)(x-1)}$ $\quad x \neq 1, -1$

- $f'(x) = \dfrac{-2x}{(x^2-1)^2}$ $\quad x \neq 1, -1$ $\quad$ Critical point: 0

Intervals of increase/decrease:

	$-\infty$ to -1	-1 to 0	0 to 1	1 to ∞
Sign of f':	+	+	−	−
behavior of f:	inc	inc	dec	dec

So 0 is a local max by 1st derivative test

- $f''(x) = \dfrac{2(x^2-1)(3x^2+1)}{(x^2-1)^4} = \dfrac{2(3x^2+1)}{(x^2-1)^3}$ $\quad x \neq 1, -1$

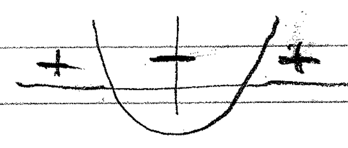
Inflection pts: None

Concavity

	$-\infty$ to -1	-1 to 0	0 to 1	1 to ∞
Sign(f''):	+	−	−	+
concave:	up	down	down	up

Notice $\text{Sign}(f'') = \text{sign}\left((x^2-1)^3\right) = \text{sign}(x^2-1)$

and x^2-1 looks like:



Asymptotes:

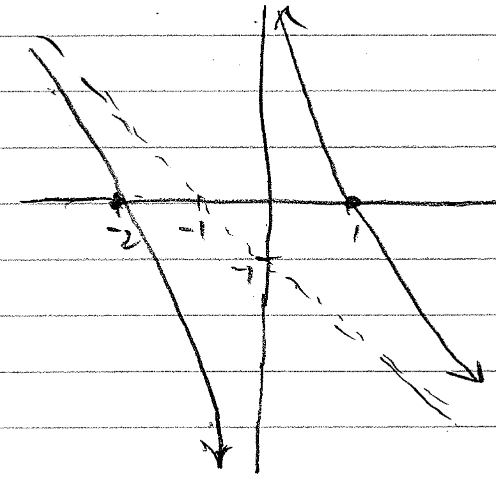
Vertical at $x=1$, $x=-1$

$\lim_{x\to 1^+} f(x) = \infty = \lim_{x\to -1^-} f(x)$, $\quad \lim_{x\to 1^-} f(x) = -\infty = \lim_{x\to -1^+} f(x)$

Since $y' < 0$, y is always decreasing
and y concave down when $x < 0$
and concave up when $x > 0$

<u>Slant asymptote</u>: $y = -(x+1)$

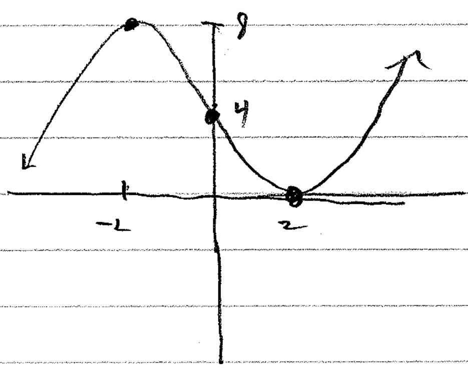
Vertical asymptote: $x = 0$



93.

	y'	y''
P	−	+
Q	+	0
R	+	−
S	0	−
T	−	−

94.



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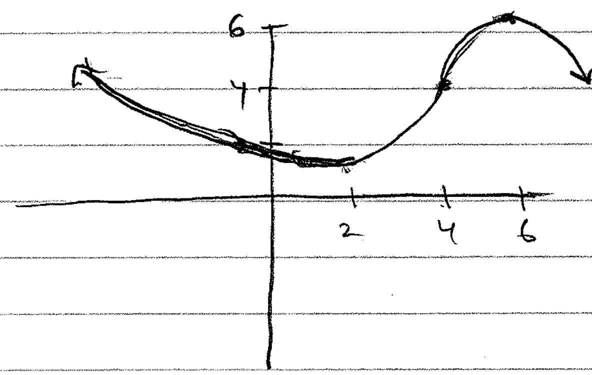

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