Homework 3

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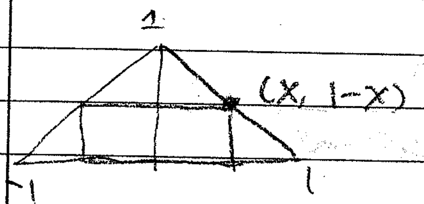
3. 

a) $y = 1 - x$

b) $A(x) = 2x(1-x) = 2x - 2x^2 \quad 0 \le x \le 1, \ (A(x) \ge 0)$

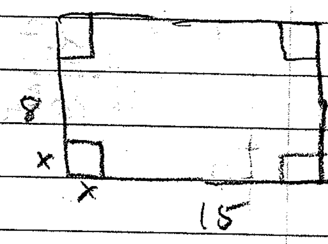
c) $A'(x) = 2 - 4x = 0$ at $x = \frac{1}{2}$

since $A(0) = A(1) = 0$, $A(\frac{1}{2}) = \frac{1}{2}$ is the maximum.

5. (Draw a picture) $\longrightarrow$

(introduce variables)

Let x be the side length of the cut out square $\underline{0 \le x \le 4}$



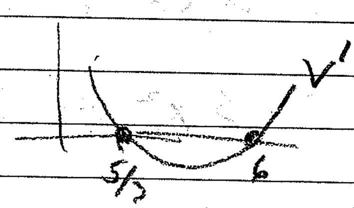
(write an equation)

$$V(x) = x(8-2x)(15-2x)$$
$$= x(4x^2 - 46x + 15 \cdot 8)$$
$$= 4x^3 - 46x^2 + 15 \cdot 8x$$
$$V'(x) = 4(3)x^2 - 2(46)x + 15 \cdot 8$$
$$= 4(3x^2 - 23x + 30)$$
$$= 4(3x-5)(x-6) = 0$$

at $5/3, 6$

only $0 \le 5/3 \le 4$, so discard 6

from the picture we conclude $5/3$ is a local max, since $V(0) = V(4) = 0$

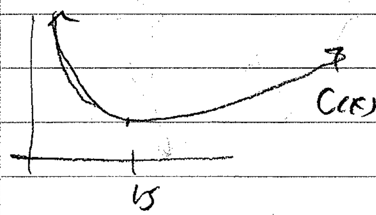


$5/3$ is a global max.

dimensions: $5/3, \ 8 - \frac{10}{3}, \ 15 - \frac{10}{3}$

$C(x) > 0$ on $(0, \infty)$ and $\lim_{x \to \infty} C(x) = \infty$

$\lim_{x \to 0^+} C(x) = \infty$ so $x = 15$ minimizes $C(x)$.

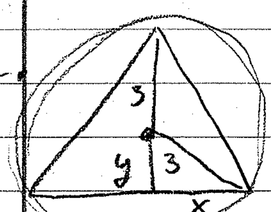


(Not to scale)

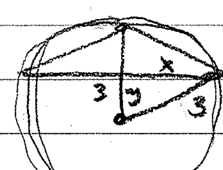
(b) The cost depends on the surface area $x^2 + 4xy$ of the submerged portion of the tank.
Making x small means each vertical face $xy = \frac{1125}{x}$ is very large and

This drives up the cost because the tank must be dug very deep.
Making x large uses up more land and presumably this is expensive to do.

12.



or



$V(x) = \frac{1}{3}\pi x^2 [y+3]$

$x^2 = 9 - y^2$

rewrite V as a function of y:

$V(y) = \frac{1}{3}\pi(9-y^2)(y+3) \qquad -3 \le y \le 3$

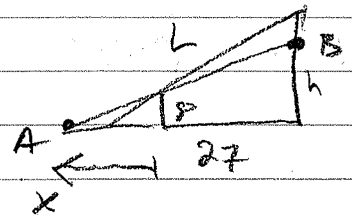
$= \frac{1}{3}\pi(27 + 9y - 3y^2 - y^3)$

$V'(y) = \frac{1}{3}\pi(9 - 6y - 3y^2) = -\pi(y^2 + 2y - 3)$

$= -\pi(y-1)(y+3) = 0$

$y = 1, -3$

Since $V(-3) = V(3) = 0$, 1 maximizes $V(y)$

39. 

let x be the distance from A to the 8' wall.
The height h from B to the ground satisfies:

(Similar triangles →) $\dfrac{h}{27+x} = \dfrac{8}{x} \implies h = \dfrac{8(27+x)}{x}$

Beam length L satisfies $L^2 = (x+27)^2 + h^2$
minimize L^2 minimizes L.

let $g(x) = (x+27)^2 + \dfrac{64}{x^2}(x+27)^2$ (simplify)

$= (x+27)^2\left[1 + \dfrac{64}{x^2}\right] \quad 0 < x < \infty$

$g'(x) = 2(x+27)\left(1+\dfrac{64}{x^2}\right) - 2(x+27)^2\left(\dfrac{64}{x^3}\right) = 0$

divide by $2(x+27) > 0$

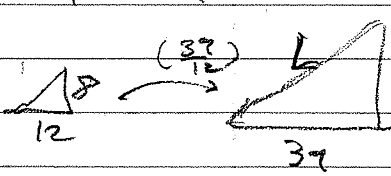
$1 + \dfrac{64}{x^2} - (x+27)\left(\dfrac{64}{x^3}\right)$ (multiply by x^3)

$x^3 + 64x - 64x - 64(27) = 0$

$x^3 = 27(64) \implies x = 3\cdot 4 = 12$

$L = \sqrt{8^2+12^2}\cdot\left(\dfrac{27+12}{12}\right)$

$= \sqrt{208}\cdot\dfrac{39}{12}$



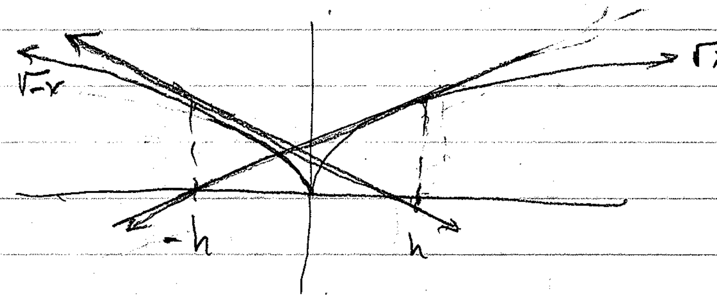
51 Cost: $C = c\,n(x)$
Revenue: $R = x\,n(x) \quad x \neq c$
Profit: $P = (x-c)n(x) = a + b(x-c)(100-x), \; x \neq c$

$P'(x) = b\left[(x-c)(-1) + (100-x)\right] = b\left[100 + c - 2x\right] = 0$
when $x = \dfrac{100+c}{2}$

§4.6

7. Suppose the first guess is a root and $f'(x_0)$ exists and is nonzero. Then the tangent line exists and intersects the x-axis also at the root x_0 since $y - 0 = f'(x_0)(x - x_0)$ is the equation for the tangent line. $y = 0$ iff $x = x_0$. Thus $x_n = x_0$ for all n.

9. $f(x) = \begin{cases} \sqrt{x}, & x \geq 0 \\ \sqrt{-x}, & x < 0 \end{cases}$



$f'(x) = \begin{cases} \frac{1}{2\sqrt{x}} & x > 0 \\ \frac{-1}{2\sqrt{-x}} & x < 0 \end{cases}$

if $x_0 = h \Longrightarrow x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = h - \frac{f(h)}{f'(h)} = h - \frac{\sqrt{h}}{\frac{1}{2\sqrt{h}}} =$

$$= h - 2(\sqrt{h})^2 = -h$$

if $x_0 = -h < 0$, Then $x_1 = -h - \frac{f(-h)}{f'(-h)} = -h - \frac{\sqrt{-(-h)}}{\left(\frac{-1}{2\sqrt{-(-h)}}\right)} =$

$$= -h + 2(\sqrt{h})^2 = h$$

So Newton's method oscillates between h and $-h$ and never converges!

§4.7 8-16 even, 20-38 even, 56 61 98

8) (a) $\frac{4}{3}\sqrt[3]{x} \longrightarrow x^{4/3}$

b) $\frac{1}{3}x^{-1/3} \longrightarrow \frac{1}{2}x^{2/3}$

c) $x^{1/3}+x^{-1/3} \longrightarrow \frac{3}{4}x^{4/3} + \frac{3}{2}x^{2/3}$

10) a) $\frac{1}{2}x^{-\frac{1}{2}} \to x^{\frac{1}{2}}$

b) $-\frac{1}{2}x^{-3/2} \longrightarrow x^{-\frac{1}{2}}$

c) $-3/2\, x^{-5/2} \longrightarrow x^{-3/2}$

12) a) $\pi\cos\pi x \longrightarrow \sin\pi x$

b) $\frac{\pi}{2}\cos\frac{\pi}{2}x \longrightarrow \sin\frac{\pi}{2}x$

c) $\cos\frac{\pi}{2}x + \pi\cos x \longrightarrow$

$\frac{2}{\pi}\sin\frac{\pi}{2}x + \pi\sin x$

14) a) $\csc^2 x$: $(\cot x)' = \left(\frac{\cos x}{\sin x}\right)' = \frac{\sin x(-\sin x) - \cos x\cos x}{\sin^2 x}$

$= \frac{-[\sin^2 x + \cos^2 x]}{\sin^2 x} = \frac{-1}{\sin^2 x} = -\csc^2 x$

so $-\cot x$

b) $-\frac{3}{2}\csc^2(\frac{3}{2}x)$. Try $\left[-\cot(\frac{3}{2}x)\right]' = -\frac{3}{2}\cot'(\frac{3}{2}x)$

$= -\frac{3}{2}\csc^2(\frac{3}{2}x)$

c) $1-8\csc^2 2x \longrightarrow x + 4\cot(2x)$

16) a) $(\sec x)' = \left(\frac{1}{\cos x}\right)' = \frac{\sin x}{\cos^2 x} = \frac{\sin x}{\cos x}\cdot\frac{1}{\cos x} = \tan x\sec x$

b) $\left(\frac{4}{3}\sec 3x\right)' = \frac{4}{3}(\sec 3x)' = \frac{4}{3}(\sec 3x\tan 3x)\cdot 3$ ✓

c) $\left(\frac{2}{\pi}\sec\frac{\pi}{2}x\right)' = \sec\pi/2\, x \tan\pi/2\, x$

28) $\int \frac{\sqrt{x}}{2} + \frac{2}{\sqrt{x}}\,dx = \int \frac{1}{2}x^{\frac{1}{2}}dx + \int 2x^{-\frac{1}{2}}dx$

$= (\frac{2}{3})\frac{1}{2}x^{3/2} + (2)2x^{1/2} + C$

$= \boxed{\frac{1}{3}x^{3/2} + 4x^{1/2} + C}$

98. (See Example 5)

Velocity $v(t)$ satisfies $v'(t) = a$ is constant

Thus $v(t) = at + C_0$

unknown, but fixed

Know that at time $t = T$, $v(T) = 0$ ft/sec

Know that position $s(T) = 45$ ft

where $s(0) = 0$ (assume this - no harm done)

Know $v(0) = 44$ ft/sec

Thus $v(0) = a \cdot 0 + C_0 = 44 \Rightarrow C_0 = 44$

So $v(t) = at + 44$

Thus since $s(t) = \frac{d}{dt} v(t)$, then

$$s(t) = \frac{a}{2} t^2 + 44t + C$$

initial position is 0; $s(0) = 0. \Rightarrow C = 0$

Final position at $t = T$ is 45: $s(T) = 45$.

Thus:

$$\boxed{\begin{aligned} s(T) &= \frac{a}{2} T^2 + 44T = 45 \\ v(T) &= aT + 44 = 0 \end{aligned}}$$

2 equations in 2 unknowns a, T.

Solve for a (or T, then a).

$$a = \frac{-44}{T} \Rightarrow \frac{-44T^2}{2T} + 44T = 22T = 45$$

$$\boxed{T = \frac{45}{22}}$$

$$\boxed{a = \frac{-44}{T} = \frac{-44(22)}{45}}$$

Verify: $v\left(\frac{45}{22}\right) = \frac{-44(22)}{45}\left(\frac{45}{22}\right) + 44 = 0$