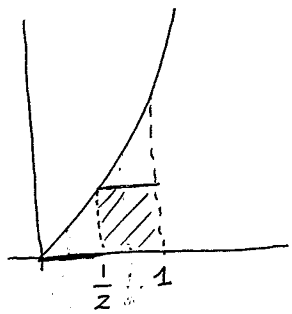
§5.1

#2

~~(a)~~



(a)

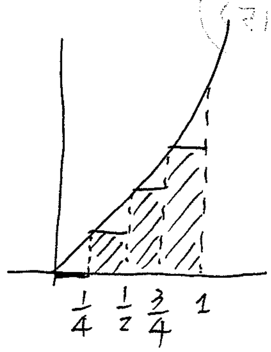
$f(0) = 0^3 = 0$

$f(\tfrac{1}{2}) = (\tfrac{1}{2}) = \tfrac{1}{4}$

Lower sum is $\frac{1}{2}(f(0) + f(\frac{1}{2})) = \frac{1}{2}(0 + \frac{1}{8}) = \frac{1}{16}$

(b) $f(\tfrac{1}{4}) = (\tfrac{1}{4})^3 = \tfrac{1}{64}$

$f(\tfrac{3}{4}) = (\tfrac{3}{4})^3 = \tfrac{27}{64}$



Lower sum is $\frac{1}{4}(f(0) + f(\frac{1}{4}) + f(\frac{1}{2}) + f(\frac{3}{4}))$

$= \frac{1}{4}(0 + \frac{1}{64} + \frac{1}{8} + \frac{27}{64})$

$= \frac{9}{64}$

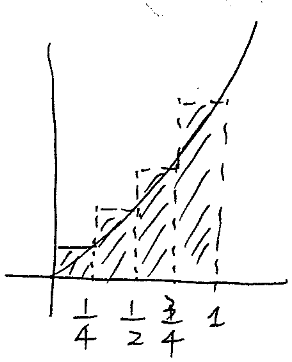
(c)

Upper sum is $\frac{1}{2}(f(\frac{1}{2}) + f(1))$

$= \frac{1}{2}(\frac{1}{8} + 1)$

$= \frac{9}{16}$

(d) Upper sum is

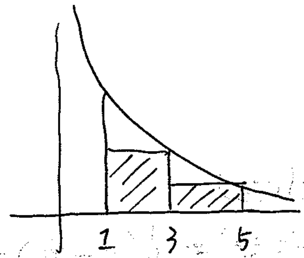


$\frac{1}{4}(f(\frac{1}{4}) + f(\frac{1}{2}) + f(\frac{3}{4}) + f(1))$

$= \frac{1}{4}(\frac{1}{64} + \frac{1}{8} + \frac{27}{64} + 1)$

$= \cancel{4}\ \frac{25}{64}$

#3

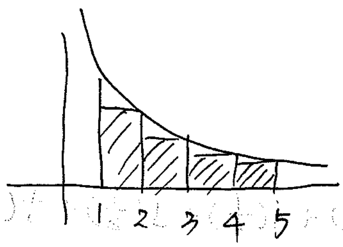


(a)

$$2\left(f(3)+f(5)\right)$$

$$= 2\left(\frac{1}{3}+\frac{1}{5}\right)$$

$$= \frac{16}{15}$$

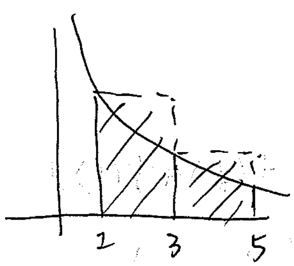


(b)

$$1\left(f(2)+f(3)+f(4)+f(5)\right)$$

$$= 1\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}\right)$$

$$= \frac{77}{60}$$

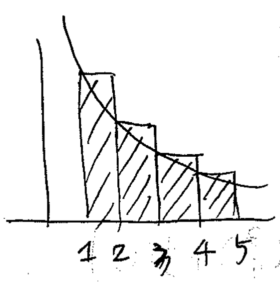


(c)

$$2\ \left(f(1)+f(3)\right)$$

$$= 2\left(\frac{1}{1}+\frac{1}{3}\right)$$

$$= \frac{8}{3}$$



(d)

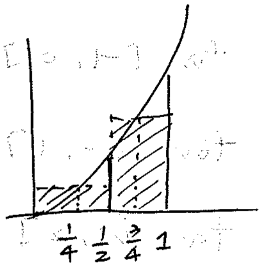
$$1\left(f(1)+f(2)+f(3)+f(4)\right)$$

$$= \frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}$$

$$= \frac{25}{12}$$

#6 $f(x) = x^3$ between $[0, 1]$

(a)



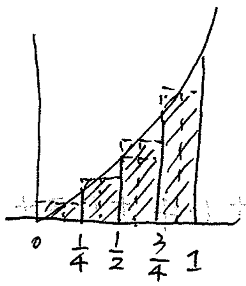
Midpoints are $\frac{1}{4}$ for $[0, \frac{1}{2}]$

$\frac{3}{4}$ for $[\frac{1}{2}, 1]$.

$$\therefore \frac{1}{2}\left(f(\tfrac{1}{4}) + f(\tfrac{3}{4})\right) = \frac{1}{2}\left(\frac{1}{64} + \frac{27}{64}\right)$$

$$= \frac{14}{64} = \frac{7}{32}.$$

(b)



Midpoints are $\frac{1}{8}$ for $[0, \frac{1}{4}]$

$\frac{3}{8}$ for $[\frac{1}{4}, \frac{1}{2}]$

$\frac{5}{8}$ for $[\frac{1}{2}, \frac{3}{4}]$

$\frac{7}{8}$ for $[\frac{3}{4}, 1]$

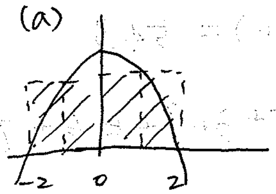
$$\therefore \frac{1}{4}\left(f(\tfrac{1}{8}) + f(\tfrac{3}{8}) + f(\tfrac{5}{8}) + f(\tfrac{7}{8})\right)$$

$$= \frac{1}{4}\left(\frac{1}{512} + \frac{27}{512} + \frac{125}{512} + \frac{343}{512}\right)$$

$$= \frac{124}{512} = \frac{31}{128}$$

#8 $f(x) = 4 - x^2$ between $[-2, 2]$

(a)



Midpoints are -1 for $[-2, 0]$

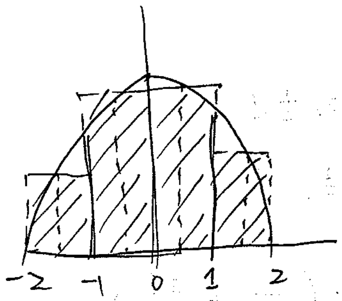
1 for $[0, 2]$

$$\therefore 2\left(f(-1) + f(1)\right)$$

$$= 2(3 + 3)$$

$$= 12$$

(b)



Midpoints are $-\frac{3}{2}$ for $[-2,-1]$

$-\frac{1}{2}$ for $[-1,0]$

$\frac{1}{2}$ for $[0,1]$

$\frac{3}{2}$ for $[1,2]$

$$1\left(f(-\tfrac{3}{2})+f(-\tfrac{1}{2})+f(\tfrac{1}{2})+f(\tfrac{3}{2})\right)$$

$= \cdot 11$

#10 (a) $5^{\times 60}(1+1.2+1.7+2.0+1.8+1.6+1.4+1.2+1.0$
$+1.8+1.5+1.2)$

$= \quad m$

(b) $5\times 60(1.2+1.7+2.0+1.8+1.6+1.4+1.2+1.0$
$+1.8+1.5+1.2+0)$

$= 4920(m)$

#19 (a) Upper : $1(70+97+136+190+265) = 758$

Lower : $1(50+70+97+136+190) = 543$

(b) Upper : $1(70+97+136+190+265+369+516+720)$

$= 2363$

Lower : $1(50+70+97+136+190+265+369+516)$

$= 1693$

(c) Best

$$\frac{25000 - 1693}{720} \approx 32.4$$

Worst

$$\frac{25000 - 2363}{720} \approx 31.4$$

§5.2

#2

$$\sum_{k=1}^{3} \frac{k-1}{k} = \frac{1-1}{1} + \frac{2-1}{2} + \frac{3-1}{3} = 0 + \frac{1}{2} + \frac{2}{3} = \frac{7}{6}$$

#8

$$1 - 2 + 4 - 8 + 16 - 32$$
$$= 1 + (-1)2^1 + 2^2 + (-1)2^3 + 2^4 + (-1)2^5$$
$$= \sum_{k=0}^{5} (-1)^k 2^k$$

For (a). $\sum_{k=1}^{6} (-2)^{k-1} = (-2)^0 + (-2)^1 + (-2)^2 + (-2)^3 + (-2)^4 + (-2)^5$

$$= 1 - 2 + 4 - 8 + 16 - 32$$

For (c). $\sum_{k=-2}^{3} (-1)^{k+1} 2^{k+2} = (-1)^{-1} 2^0 + (-1)^0 2^1 + (-1)^1 2^2 + (-1)^2 2^3 + (-1)^3 2^4 + (-1)^4 2^5$

$$= -1 + 2 - 4 + 8 - 16 + 32$$

∴ Answers are (a) and (b).

#10

(a) $\sum_{k=1}^{4} (k-1)^2 = 0^2 + 1^2 + 2^2 + 3^2$

(b) $\sum_{k=1}^{3} (k+1)^2 = 0^2 + 1^2 + 2^2 + 3^2 + 4^2$

(c) $\sum_{k=-3}^{-1} k^2 = (-3)^2 + (-2)^2 + (-1)^2 = 1^2 + 2^2 + 3^2 = (a)$

Answer is (b).

#20 (a) $\sum_{k=1}^{13} k = 1 + 2 + \cdots + 13 = \frac{13(13+1)}{2} = \frac{13 \times \cancel{14}^{7}}{\cancel{2}} = 91$

(b) $\sum_{k=1}^{13} k^2 = 1^2 + 2^2 + \cdots + 13^2 = \frac{13(13+1)(2\cdot 13+1)}{6} = \frac{13\cdot 14\cdot 27}{6} = 819$

(c) $\sum_{k=1}^{13} k^3 = 1^3 + 2^3 + \cdots + 13^3 = \left[\frac{13(13+1)}{2}\right]^2 = \left(\frac{13 \times \cancel{14}^{7}}{\cancel{2}}\right)^2 = 91 \times 91 = 8281$

#26

$$\sum_{k=1}^{7} k(2k+1) = \sum_{k=1}^{7} 2k^2+k$$

$$= \sum_{k=1}^{7} 2k^2 + \sum_{k=1}^{7} k \quad \text{(by sum rule)}$$

$$= 2\sum_{k=1}^{7} k^2 + \sum_{k=1}^{7} k \quad \text{(by constant multiple rule)}$$

$$= 2\cdot\frac{7(7+1)(2\cdot 7+1)}{6} + \frac{7(7+1)}{2}$$

$$= 280 + 28$$

$$= 308$$

#31 (a) $$\sum_{k=1}^{n} 4 = \overbrace{4+4+\cdots+4}^{n \text{ times}} = 4n$$

(b) $$\sum_{k=1}^{n} c = c+c+\cdots+c = cn$$

(d) $$\sum_{k=1}^{n} (k-1) = \sum_{k=1}^{n} k - \sum_{k=1}^{n} 1 \quad \text{(by difference rule)}$$

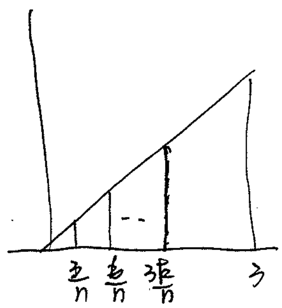
$$= \frac{n(n+1)}{2} - n$$

$$= \frac{n^2+n}{2} - \frac{2n}{2}$$

$$= \frac{n^2+n-2n}{2}$$

$$= \frac{n^2-n}{2}$$

#40



$[0,3]$

Divide into n equal subintervals of length $\frac{3}{n}$

$\Rightarrow \quad c_k = \frac{3}{n} \cdot k = \frac{3k}{n}$

Therefore,

$$S_p = \sum_{k=1}^{n} f(c_k) \cdot \frac{3}{n}$$

$$= \sum_{k=1}^{n} (2c_k) \cdot \frac{3}{n}$$

$$= \sum_{k=1}^{n} \frac{6k}{n} \cdot \frac{3}{n}$$

$$= \sum_{k=1}^{n} \frac{18k}{n^2}$$

$$= \frac{18}{n^2} \sum_{k=1}^{n} k$$

(by constant multiple rule and $\frac{18}{n^2}$ is a constant).

$$= \frac{\cancel{18}^{\,9}}{n^{\cancel{2}}} \cdot \frac{\cancel{n}(n+1)}{\cancel{2}}$$

$$= \frac{9(n+1)}{n} = \frac{9n+9}{n}$$

$$= 9 + \frac{9}{n}$$

$$\Rightarrow \lim_{n\to\infty} S_p = \lim_{n\to\infty} 9 + \frac{9}{n} = 9.$$

P 7.2

#5 $200+195+190+\cdots+70+65$

$b=200,\ d=-5,\quad b+(n-1)d=65$

$\Rightarrow 200+(n-1)(-5)=65$

$\Rightarrow 205-5n=65$

$n=28.$

$28\left(200+\frac{(28-1)(-5)}{2}\right)=28\left(200-\frac{135}{2}\right)$

$=5600-14\cdot 135$

$=3710.$

#7 $\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\cdots+\frac{1}{4^{50}}$

$b=\frac{1}{4},\ r=\frac{1}{4},\quad br^{n-1}=\frac{1}{4^{50}}$

$\Rightarrow \frac{1}{4}\left(\frac{1}{4}\right)^{n-1}=\frac{1}{4^{50}}$

$\Rightarrow \left(\frac{1}{4}\right)^{n}=\frac{1}{4^{50}},\ n=50$

$\frac{1}{4}+\frac{1}{16}+\cdots+\frac{1}{4^{50}}=\frac{1}{4}\,\frac{1-\left(\frac{1}{4}\right)^{50}}{1-\frac{1}{4}}$

$=\frac{1}{4}\,\frac{1-\left(\frac{1}{4}\right)^{50}}{3/4}$

$=\frac{1}{3}\left(1-\frac{1}{4^{50}}\right).$

#10

$$\sum_{k=5}^{65}(4k-1) = 19+23+27+\cdots+259$$

$b=19,\ d=4,\ n=61.$

$$61\left(19+\frac{(61-1)4}{2}\right) = 61\left(19+\overset{120}{}\right)$$

$$= 61\cdot 139$$

$$= 8479$$

#12 $1001+1003+\cdots+9999$

$b=1001,\ d=2,\quad b+(n-1)d=9999$

$$1001+(n-1)2=9999$$

$$n=4500$$

$$4500\left(1001+\frac{(4500-1)2}{2}\right)$$

$$= 4500\cdot 5500$$

$$= 44750000$$

#13 $1003+1013+1023+\cdots+9993$

$b=1003,\ d=10,\ 1003+(n-1)10=9993$

$$n=900$$

$$900\left(1003+\frac{(900-1)10}{2}\right)$$

$$= 900\cdot 5498$$

$$= 4948200$$

#16 $1+2+4+\cdots+2^{100}$

$$= 1\cdot\frac{1-2^{101}}{1-2} = 2^{101}-1$$

#18 $\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\cdots+\frac{1}{3^{33}}$

$$= \frac{1}{3}\cdot\frac{1-\left(\frac{1}{3}\right)^{33}}{1-\frac{1}{3}} = \frac{1}{2}\left(1-\left(\frac{1}{3}\right)^{33}\right)$$

#20 $1-\frac{1}{3}+\frac{1}{9}-\frac{1}{27}+\cdots+\frac{1}{3^{62}}$

$$= 1\cdot\frac{1-\left(-\frac{1}{3}\right)^{63}}{1-\left(-\frac{1}{3}\right)} = \frac{3}{4}\left(1-\left(-\frac{1}{3}\right)^{63}\right)$$

$$= \frac{3}{4}\left(1+\frac{1}{3^{63}}\right)$$

#35 $b+(b+d)+\cdots+(b+(n-1)d)$

$$= \sum_{m=1}^{n}\left[b+(m-1)d\right]$$

$$= n\left(b+\frac{(n-1)d}{2}\right)$$

#36 $b+br+\cdots+br^{n-1}$

$$= \sum_{m=1}^{n} br^{m-1}$$

$$= b\,\frac{1-r^n}{1-r}.$$