§5.4

#6. $\int_{-2}^{2}(x^3-2x+3)\,dx = \frac{1}{4}x^4 - x^2 + 3x \Big|_{-2}^{2}$

$= \left(\frac{1}{4}\cdot 2^4 - 2^2 + 3\cdot 2\right) - \left(\frac{1}{4}(-2)^4 - (-2)^2 + 3\cdot(-2)\right)$

$= (4-4+6) - (4-4-6)$

$= 12.$

#8 $\int_{1}^{32} x^{-6/5}\,dx = \frac{1}{-\frac{1}{5}} x^{-\frac{1}{5}} \Big|_{1}^{32} = -5x^{-\frac{1}{5}}\Big|_{1}^{32}$

$= \left(-5\cdot 32^{-\frac{1}{5}}\right) - \left(-5\cdot 1^{-\frac{1}{5}}\right)$

$= -5\cdot\frac{1}{\sqrt[5]{32}} - (-5)$

$= -\frac{5}{2} + 5 = \frac{5}{2}$

#10 $\int_{0}^{\pi}(1+\cos x)\,dx = x + \sin x\Big|_{0}^{\pi}$

$= (\pi + \sin\pi) - (0 + \sin 0)$

$= \pi.$

#12 $\int_{0}^{\pi/3} 4\sec u\tan u\,du = 4\sec u\Big|_{0}^{\frac{\pi}{3}}$

$= 4\sec\frac{\pi}{3} - 4\sec 0$

$= 8-4$

$= 4$

#14 $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1-\cos 2t}{2}\,dt = \frac{1}{2}t - \frac{\sin 2t}{4}\Big|_{-\frac{\pi}{3}}^{\frac{\pi}{3}}$

$= \left(\frac{1}{2}\cdot\frac{\pi}{3} - \frac{\sin\frac{2\pi}{3}}{4}\right) - \left(\frac{1}{2}\left(-\frac{\pi}{3}\right) - \frac{\sin\left(-\frac{2\pi}{3}\right)}{4}\right)$

$= \frac{\pi}{3} + \frac{\sqrt{3}}{4}.$

#16 $\int_0^{\frac{\pi}{6}} (\sec x+\tan x)^2\,dx = \int_0^{\frac{\pi}{6}} (\sec^2 x + 2\sec x\tan x + \tan^2 x)\,dx$

$= \int_0^{\frac{\pi}{6}} \left[\sec^2 x + 2\sec x\tan x + (\sec^2 x - 1)\right] dx$

$= \int_0^{\frac{\pi}{6}} (2\sec^2 x + 2\sec x\tan x - 1)\,dx$

$= 2\tan x + 2\sec x - x \Big|_0^{\frac{\pi}{6}}$

$= \left(2\tan\frac{\pi}{6} + 2\sec\frac{\pi}{6} - \frac{\pi}{6}\right) - (2\tan 0 + 2\sec 0 - 0)$

$= \left(\frac{2}{\sqrt{3}} + \frac{4}{\sqrt{3}} - \frac{\pi}{6}\right) - (0+2-0)$

$= 2\sqrt{3} - \frac{\pi}{6} - 2$

#18 $\int_{-\frac{\pi}{3}}^{-\frac{\pi}{4}} \left(4\sec^2 t + \frac{\pi}{t^2}\right) dt = \int_{-\frac{\pi}{3}}^{-\frac{\pi}{4}} (4\sec^2 t + \pi t^{-2})\,dt$

$= 4\tan t + \frac{\pi}{-1} t^{-1} \Big|_{-\frac{\pi}{3}}^{-\frac{\pi}{4}}$

$= \left(4\tan\left(-\frac{\pi}{4}\right) - \frac{\pi}{-\frac{\pi}{4}}\right) - \left(4\tan\left(-\frac{\pi}{3}\right) - \frac{\pi}{-\frac{\pi}{3}}\right)$

$= (-4+4) - (-4\sqrt{3}+3)$

$= 4\sqrt{3} - 3.$

#20 $\int_{-\sqrt{3}}^{\sqrt{3}} (t+1)(t^2+4)\,dt = \int_{-\sqrt{3}}^{\sqrt{3}} (t^3 + t^2 + 4t + 4)\,dt$

$= \frac{1}{4}t^4 + \frac{1}{3}t^3 + 2t^2 + 4t \Big|_{-\sqrt{3}}^{\sqrt{3}}$

$= \left(\frac{1}{4}(\sqrt{3})^4 + \frac{1}{3}(\sqrt{3})^3 + 2(\sqrt{3})^2 + 4\sqrt{3}\right)$

$- \left(\frac{1}{4}(-\sqrt{3})^4 + \frac{1}{3}(-\sqrt{3})^3 + 2(-\sqrt{3})^2 + 4(-\sqrt{3})\right)$

$= \left(\frac{9}{4} + \sqrt{3} + 6 + 4\sqrt{3}\right) - \left(\frac{9}{4} - \sqrt{3} + 6 - 4\sqrt{3}\right)$

$= 10\sqrt{3}$

#22 $\int_{-3}^{-1} \frac{y^5 - 2y}{y^3}\, dy = \int_{-3}^{-1} (y^2 - 2y^{-2})\, dy$

$= \frac{1}{3} y^3 - \frac{2}{-1} y^{-1} \Big|_{-3}^{-1}$

$= \frac{1}{3} y^3 + 2y^{-1} \Big|_{-3}^{-1}$

$= \left(\frac{1}{3}(-1)^3 + 2/-1\right) - \left(\frac{1}{3}(-3)^3 + 2/-3\right)$

$= \left(-\frac{1}{3} - 2\right) - \left(-9 - \frac{2}{3}\right)$

$= 7 + \frac{1}{3}$

$= \frac{22}{3}.$

#30

(a). $\int_1^{\sin x} 3t^2\, dt = t^3 \Big|_1^{\sin x} = (\sin x)^3 - 1$

$\frac{d}{dx} \int_1^{\sin x} 3t^2\, dt = \frac{d}{dx}\left((\sin x)^3 - 1\right)$

$= 3(\sin x)^2 \cdot \cos x.$

(b) $\frac{d}{dx} \int_1^{\sin x} 3t^2\, dt = 3(\sin\ )^2 \cdot (\sin x)'$

$= 3(\sin x)^2 \cdot \cos x$

#32

(a). $\int_0^{\tan\theta} \sec^2 y\, dy = \tan y \Big|_0^{\tan\theta} = \tan(\tan\theta) - \tan 0$

$= \tan(\tan\theta)$

$\frac{d}{d\theta} \int_0^{\tan\theta} \sec^2 y\, dy = \frac{d}{d\theta}\left(\tan(\tan\theta)\right)$

$= \sec^2(\tan\theta) \cdot (\tan\theta)'$

$= \sec^2(\tan\theta) \cdot \sec^2\theta.$

(b) $\frac{d}{d\theta} \int_0^{\tan\theta} \sec^2 y\, dy = \sec^2(\tan\theta) \cdot (\tan\theta)'$

$= \sec^2(\tan\theta) \cdot \sec^2\theta.$

#36.

$$\frac{dy}{dx} = \frac{d}{dx}\left(x \int_2^{x^2} (\sin t^3)\, dt \right)$$

$$= \int_2^{x^2} (\sin t^3)\, dt + x \cdot \frac{d}{dx} \int_2^{x^2} \sin t^3 \, dt$$

$$= \int_2^{x^2} \sin t^3 \, dt + x \cdot \sin (x^2)^3 \cdot (x^2)'$$

$$= \int_2^{x^2} \sin t^3 \, dt + 2x^2 \sin x^6 .$$

#37.

$$\frac{dy}{dx} = \frac{d}{dx}\left(\int_{-1}^{x} \frac{t^2}{t^2+4}\, dt - \int_{3}^{x} \frac{t^2}{t^2+4}\, dt \right)$$

$$= \frac{d}{dx}\left(\int_{-1}^{3} \frac{t^2}{t^2+4}\, dt \right)$$

$= 0$ since $\int_{-1}^{3} \frac{t^2}{t^2+4}\, dt$ is a constant function.

#40

$$\frac{dy}{dx} = \frac{d}{dx} \int_{\tan x}^{0} \frac{dt}{1+t^2}$$

$$= \frac{d}{dx}\left(- \int_0^{\tan x} \frac{dt}{1+t^2} \right)$$

$$= - \frac{1}{1+(\tan x)^2} \cdot (\tan x)'$$

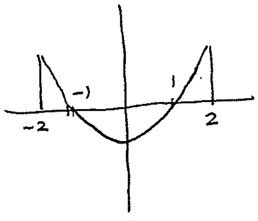
$$= - \frac{1}{1+\tan^2 x} \cdot \sec^2 x$$

$$= - \frac{1}{\sec^2 x} \cdot \sec^2 x$$

$$= -1 .$$

#42

$3x^2-3=0.$ $\quad x=\pm 1.$



Area between $y=3x^2-3$ and x-axis is

$$\int_{-2}^{-1}(3x^2-3)\,dx+\int_{-1}^{1}-(3x^2-3)\,dx+\int_{1}^{2}(3x^2-3)\,dx$$

$$= x^3-3x\Big|_{-2}^{-1}+(-x^3+3x)\Big|_{-1}^{1}+(x^3-3x)\Big|_{1}^{2}$$

$$=\left(2-(-2)\right)+\left((-1+3)-(-2)\right)+\left((2)-(-2)\right)$$

$$=\cancel{0}\,4+4+4$$

$$=\cancel{8}.\,12$$

#46

$\sin\frac{\pi}{6}=\frac{1}{2}.$

$$\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}\left(\sin x-\frac{1}{2}\right)dx=-\cos x-\frac{1}{2}x\Big|_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$$

$$=\left(-\cos\frac{5\pi}{6}-\frac{1}{2}\cdot\frac{5\pi}{6}\right)-\left(-\cos\frac{\pi}{6}-\frac{1}{2}\cdot\frac{\pi}{6}\right)$$

$$=\left(\frac{\sqrt{3}}{2}-\frac{5\pi}{12}\right)-\left(-\frac{\sqrt{3}}{2}-\frac{\pi}{12}\right)$$

$$=\sqrt{3}-\frac{\pi}{3}$$

#62

$$\int_0^x f(t)\,dt=x\cos\pi x.$$

$$\frac{d}{dx}\int_0^x f(t)\,dt=\frac{d}{dx}\left(x\cos\pi x\right)$$

$$f(x)=\cos\pi x+x\cdot(-\sin\pi x)\cdot\pi$$

$$=\cos\pi x-\pi x\sin\pi x$$

$$\therefore f(4)=\cos(4\pi)-4\pi\cdot\sin(4\pi)$$

$$=1.$$

#64 $g'(x) = \frac{d}{dx}\left(3 + \int_1^{x^2} \sec(t-1)\,dt\right)$

$= 0 + \sec(x^2-1)\cdot 2x$

$= 2x\sec(x^2-1)$

$g'(-1) = 2(-1)\sec((-1)^2-1)$

$= -2\sec 0$

$= -2.$

$g(-1) = 3 + \int_1^1 \sec(t-1)\,dt = 3 + 0 = 3$

$\therefore\ g(x) \approx \cancel{3+g'}\ g(-1) + g'(-1)(x-(-1))$

$= 3 + (-2)(x+1)$

$= 3 - 2x - 2$

$= 1 - 2x.$