THINGS TO LOOK OUT FOR WHEN YOU WRITE A SOLUTION TO A LIMIT PROBLEM

Problem 23.1. Find the derivative of $f(x) = x^2 - x$.
One has

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{\big[(x+h)^2-(x+h)\big]-\big[x^2-x\big]}{h} \\ &= \lim_{h\to 0} \frac{2xh+h^2-h}{h} \\ &= \lim_{h\to 0} 2x+h-1 \\ &= 2x-1. \end{aligned}$$

If you don't like writing "$\lim_{h\to 0}$" all the time , then you could write this:

We want to find

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}.$$

Now simplify the quantity in the limit:

$$\begin{aligned} \frac{\big[(x+h)^2-(x+h)\big]-\big[x^2-x\big]}{h} &= \frac{2xh+h^2-h}{h} \\ &= 2x+h-1 \end{aligned}$$

Hence

$$f'(x) = \lim_{h\to 0} 2x+h-1 = 2x-1.$$

In contrast to the two good solutions above, the following is bad notation and will only get half credit (at best):

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \frac{\big[(x+h)^2-(x+h)\big]-\big[x^2-x\big]}{h} \\ &= \frac{2xh+h^2-h}{h} \\ &= 2x+h-1 \\ &= 2x-1. \end{aligned}$$

The notation is bad, because several of the equations here aren't true, or don't make sense. For instance, the last two lines say " $2x+h-1 = 2x-1$" and that can only be true when $h = 0$. That is not what we wanted. The quantity h is something that approaches zero, without ever becoming zero. Moreover h is a "dummy variable." Read §17 of the notes to see what that means.