More problems on integration by parts

1 Compute the following integrals, using integration by parts (and an occasional substitution). For the definite integrals draw the integrand (the function being integrated), and interpret the integral in terms of the area of a region in the plane, or a combination of such areas.

\[(i) \int (2x - 1)e^x \, dx \quad (ii) \int x \ln(3x + 2) \, dx \quad (iii) \int (12s^2 - 4s)e^{-2s} \, ds \]

\[(iv) \int p^3 \cos p^2 \, dp \quad (v) \int x^2 \sin \frac{x}{2} \, dx \quad (vi) \int (3x^2 + 2x) \ln(x + 1) \, dx \]

\[(vii) \int \arctan x \, dx \quad (viii) \int 2t \arctan t \, dt \quad (ix) \int \theta \cos(2\theta + 4) \, d\theta \]

\[(x) \int \ln(t^2 + 1) \, dt \quad (xi) \int_0^1 (x - x^3) \cos \pi x \, dx \quad (xii) \int_1^e \ln \rho \, d\rho \]

\[(xiii) \int_{-1}^1 (1 - z^2)e^z \, dz \quad (xiv) \int_0^a (a - x)^3 e^{-x} \, dx \quad (xv) \int_0^a z(a^2 - z^2)^3 e^{-z^2} \, dz \]

(a is a positive constant.)

2 Prove the formula for partial integration (without looking at the notes), i.e. prove this:

\[\int_a^b f(x)g'(x) \, dx = [f(x)g(x)]_a^b - \int_a^b f'(x)g(x) \, dx.\]

3 Let \(y = f(x)\) be a strictly increasing function which is defined for \(a \leq x \leq b\), and let \(x = g(y)\) be its inverse.

(i) Show that

\[\int_a^b x f'(x) \, dx + \int_a^b f(x) \, dx = [xf(x)]_a^b\]

(ii) Show that

\[\int_{f(a)}^{f(b)} g(y) \, dy = \int_a^b x f'(x) \, dx.\]

(hint: use a substitution.)

(iii) Explain that

Area 1 + Area 2 = \([xf(x)]_a^b\)

is a different way of writing the formula from problem (i).