QUESTIONS FROM SOME OLD SECOND MIDTERMS

MATH 222, LECTURE 3, SPRING 2008

(1) Compute $\lim_{x\to 0} \dfrac{e^{x^2}\cos x - 1 - \frac{1}{2}x^2}{1-\cos(x^2)}$
[Hint: For partial credit first compute the Taylor expansions up to $o(x^4)$ of $e^{x^2}\cos x$ and of $\cos(x^2)$.]

(2) Show that the Taylor series for
$$f(x) = \sin(3x+2)$$
converges for *all* real numbers x.

(3) Find
$$\mathbf{Re}\left\{\frac{e^{(1+i)x}}{2+i}\right\} \quad \text{and} \quad \mathbf{Re}\left\{\frac{2+i}{e^{(1+i)x}}\right\}$$

(4) You are given an angle x whose Cosine and Sine are given by
$$\cos x = A, \text{ and } \sin x = \sqrt{1-A^2}.$$
Compute $\cos 5x$.

(5) (a) Compute the Taylor expansion up to $o(x^7)$ of $f(x) = \ln(1+x^3) - x\sin x^2$.
(b) Compute the following limits
$$A = \lim_{x\to 0}\frac{f(x)}{x^5}, \quad B = \lim_{x\to 0}\frac{f(x)}{x^6}, \quad C = \lim_{x\to 0}\frac{f(x)}{x^7}.$$
where $f(x)$ is as above.

(6) Show that the Taylor series for
$$f(x) = e^{-2x^4}$$
converges to $f(x)$ for *all* real numbers x.

(7) Consider the Taylor series
$$f(x) = \frac{1+x}{(2+x)(3-x)} = f_0 + f_1 x + f_2 x^2 + \cdots + f_n x^n + \cdots$$
If $f_{4931} = A$ and $f_{4932} = B$ then compute f_{4933} and f_{4934}.

(8) Let $z = 1 + i\sqrt{3}$.
(a) Find the smallest integer $n > 0$ for which z^n is a negative real number.
(b) Find the smallest integer $n > 0$ for which z^n is a positive real number.
(c) Same as above if $z = -\sqrt{3} + i$.
(d) Same as above if $z = -2 - 2i$.
(e) Find all complex numbers z for which $z^2 = -i$.
(f) Find all complex numbers z for which $z^3 = i$.
(g) Find all complex numbers z for which $z^2 = 1 + i$.