**First Midterm Exam**

Name:

Begin each problem on a new page. Number your pages. At the end of the exam staple all pages in order together, with this page as cover.

All answers should be explained. Scrap paper is available in large quantities.

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**Problem 1** Prove by induction that the numbers $A_n$ given by

$$A_n = 1 + 4 + 7 + 10 + \cdots + (3n - 2)$$

satisfy

$$A_n = \frac{3}{2} n^2 - \frac{1}{2} n$$

for all $n = 1, 2, 3, \ldots$

**Problem 2** Use the $\varepsilon$-$\delta$ definition of $\lim_{x \to a} f(x)$ to do these two problems.

a) Show for any number $a$ that

$$\lim_{x \to a} |x| = |a|.$$

b) Show that

$$\lim_{x \to 0} \frac{x}{4x + 1} = 0$$

**Problem 3** By definition, a function $f$ is said to be **bounded from above** on an interval $(a, b)$ if a number $M$ exists such that $f(x) \leq M$ for all $x \in (a, b)$.

a) Is $f(x) = 1/x$ bounded from above on the interval $(0, 1)$?

b) Is $g(x) = 1 - 1/x$ bounded from above on the interval $(0, 1)$?

c) Is $h(x) = x - x^2$ bounded from above on the interval $(-\infty, \infty)$?

**Problem 4** Find the least upper bound (‘supremum’) of the following sets. You should of course explain your answers.

a) $E = (0, 4)$

b) $F = (0, 4]$ 

c) $G = \left\{ \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots \right\}$ i.e. the set of all numbers of the form $\frac{n-1}{n}$, for $n = 2, 3, 4, \ldots$