PROBLEMS ABOUT LIMITS WITH & WITHOUT $\varepsilon$–$\delta$ (REDO)

Problem 3ii.

Show directly from the definition of limits that

$$\lim_{x \to a} \frac{1}{x} = \frac{1}{a}$$

for any $a \neq 0$.

[Avoid, repeat, avoid using Spivak’s problems Lemma on page 101 – “Why?” you ask. Well this is a problem you should be able to do ‘from scratch’ without memorizing the fairly complicated formulas Spivak provides.]

The following two problems have simple short answers, and are not “$\varepsilon$–$\delta$” problems. You’re not asked to (dis)prove the statements directly from the definition, so that means that you can use properties of ‘$\lim_{x \to a}$’ that were proved in chapter 5 of Spivak’s book.

Problem 8a. If $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ both do not exist, must is be true that $\lim_{x \to a} (f(x) + g(x))$ also does not exist. [If you think so, provide a proof, if you don’t think so, give a counterexample.]

Problem 8b. If $\lim_{x \to a} f(x)$ and $\lim_{x \to a} (f(x) + g(x))$ both do not exist, must is be true that $\lim_{x \to a} g(x)$ also does not exist. [If you think so, provide a proof, if you don’t think so, give a counterexample.]

Solutions should be “well written”: You should be able to read your solution out loud. Every sentence should have a subject and a verb. Avoid using symbols like ‘$\to$’ when you want to say “implies.”