

Angle-Preserving Flattening Maps and the 3D Visualization of Colon CT Images

Steven Haker, Ph. D.

Department of Electrical and Computer Engineering
University of Minnesota
Minneapolis, MN 55455

Sigurd Angenent, Ph. D.

Department of Mathematics
University of Wisconsin
Madison, Wisconsin 53705

Allen Tannenbaum*, Ph. D.

Department of Electrical and Computer Engineering
University of Minnesota
Minneapolis, MN 55455
email: tannenba@ece.umn.edu
fax: 612-625-4583

Ron Kikinis, M. D.

Harvard Medical School
Brigham and Women's Hospital
Harvard University
Boston, MA 02115

November 8, 1999

This work was supported in part by grants from the National Science Foundation DMS-9058492, ECS-99700588, NSF-LIS, by the Air Force Office of AF/F49620-98-1-0168, by the Army Research Office DAAG55-98-1-0169, and MURI Grant.

*All correspondence concerning this paper should be sent to Allen Tannenbaum.

Abstract

• **Objective:** In this paper, we consider a novel 3D visualization technique based on surface flattening for virtual colonoscopy. Such visualization methods could have a major impact in virtual colonoscopy since they have the potential for non-invasively determining the presence of polyps and other pathologies. Further, we demonstrate a method which presents a surface scan of the entire colon as a cine, and affords viewer the opportunity to exam each point on the surface without distortion.

• **Material and Methods:** We use certain angle-preserving mappings from differential geometry in order to derive an explicit method for flattening surfaces obtained from 3D colon CT imagery. Indeed, we describe a general method based on a discretization of the Laplace-Beltrami operator for flattening a surface onto the plane in a manner which preserves the local geometry. From a triangulated surface representation of the colon, we indicate how the procedure may be implemented using a finite element technique, which takes into account special boundary conditions. We also provide simple formulas which may be used in a real time cine to correct for distortion.

• **Results:** We applied our method to 3D colon CT data provided to us by the Surgical Planning Laboratory of Brigham and Women's Hospital. This example consists in a test of flattening the colon surface contained in a $256 \times 256 \times 124$ CT colon image. We initially find the colon surface from the 3D data using certain snake-based segmentation methods. We then create a triangularization of this surface using the Visualization Toolkit. Next, the angle-preserving flattening method is applied to the triangulated surface. We indicate the flattened representations both of the interior and exterior of the colon. We also indicate a few frames of a distortion correcting cine.

• **Conclusions:** The angle-preserving geometric procedure gives a way of flattening the entire colon. Locally shape is preserved. The

finite element algorithm which finds the flattening map is very fast, and indeed the whole procedure from segmentation to flattening takes about 12 minutes on a Sun Ultrasparc 10 Workstation. Adjustments to the flattening map can be made in real time to provide a viewer with a distortion correcting cine. The flattening of highly undulated surfaces has other uses in medical image processing and visualization including brain surface unfolding in functional MRI.

Key words: Virtual colonoscopy, CT colonography, flattening maps, shape preservation.

1 Introduction

3D visualization is becoming an increasingly important technique in surgical planning, non-invasive diagnosis and treatment, and image-guided surgery. Surface warping and flattening, which allow the easy visualization of highly undulated surfaces, are methods that are becoming increasingly widespread. For example, flattened representations of the brain cortical surface are essential in functional magnetic resonance imaging since one wants to show neural activity deep within the folds or sulci of the brain. 3D visualization is also of great importance in virtual colonoscopy in which one can non-invasively determine the presence of pathologies.

Virtual colonoscopy is currently an active area of research by radiologists as a minimally invasive screening method for the detection of small polyps (see [7, 8] and the references therein). In the colon, this has become possible because of imaging devices which allow single breath hold acquisitions of the entire abdomen at acceptable resolutions. Most reports have focused on methods which use computer graphics to simulate conventional colonoscopic procedures [8, 13, 16].

Virtual colonoscopy has some fundamental problems, which it shares with conventional colonoscopy. The most important one is that the navigation using inner views is very challenging and it happens frequently that sizable areas are not inspected at all, leading to incomplete examinations. An alternative approach for the inspection of the entire surface of the colon is to simulate the approach favored by pathologists, which involves cutting open the tube represented by the colon, and laying it out flat for a comprehensive inspection. In some very recent work [11], a visualization technique is pro-

posed using cylindrical and planar map projections. It is well-known that such projections can cause distortions in shape as is discussed in [11] and the references therein.

In this paper, we take another approach. We present a method for mapping the colon onto a flat surface in a conformal manner. A *conformal mapping* is a one-to-one mapping between surfaces which preserves angles, and thus preserves the local geometry as well. Our approach to flattening such a surface is based on a certain mathematical technique from Riemann surface theory, which allows us to map any highly undulating tubular surface without handles or self-intersections onto a planar rectangle in a conformal manner. There is also work in [18] on the topological flattening of a tube onto the plane and the application to virtual colonoscopy.

From a triangulated surface representation of the colon, we indicate how the procedure may be implemented using finite elements. Moreover, we explicitly show how various structures and pathologies of the colon may be studied using this approach. In contrast to virtual colonoscopy methods which are based on a “fly-through,” our method allows the entire colon surface to be viewed at once, unobscured by surface folds. Hence the flattening mapping can be used to obtain an atlas of the given surface in a straightforward, canonical manner.

The key observation is that the flattening function may be obtained as the solution of a second order elliptic partial differential equation (PDE) on the surface to be flattened. For triangulated surfaces, there exist powerful, reliable finite element procedures which can be employed to numerically approximate the flattening function. This numerical method involves merely solving two sparse systems of linear equations and can be accomplished by using standard conjugate gradient techniques.

Here we are interested in exploiting our work for the discovery of various colon pathologies. However, in addition to the colon, we have been employing our method in the study of depth maps of bladder images, and in the study of curvatures and polar maps from ultrasound heart images. We are also interested in using our work for the construction of canonical brain models and coordinates on both the grey matter and grey/white matter boundaries from 3D MR data sets [1].

We now summarize the contents of this paper. In Section 2, we give a high-level overview of our flattening method. In Section 3, we describe the approximation method we employ for numerically computing the flattening

mapping based on finite elements. In Section 4, we provide formulas which can be used in real time to create a cine which provides a view of each point of the surface without distortion. Then in Section 5, we illustrate the methodology on some CT colon imagery, and in Section 6, we draw some conclusions about our general approach as well as directions for future research. Finally in Section 7, we provide some key mathematical details justifying our methodology.

2 General Approach to Colon Flattening

We first consider a mathematical model for the colon surface. Accordingly, let $\Sigma \subset \mathbf{R}^3$ represent an embedded surface (no self-intersections) which is topologically an open-ended cylinder. In the mathematical appendix (see Section 7 below), we will give more details on the analytical basis for flattening such a surface onto the plane. In this section, we just sketch some of the key points.

We assume that Σ is a smooth manifold. For the finite element method described in the next section, it will be enough to take it as a triangulated surface. We refer the reader to [3] for the basic theory of surfaces of the type we are considering here, and to [12] for the relevant results on partial differential equations.

The boundary of Σ consists of two topological circles, which we will call σ_0 and σ_1 . We want to construct a conformal map (i.e., a map which preserves angles and is one-to-one), $f : \Sigma \rightarrow \mathbf{C}$, which sends Σ to an annulus A such that σ_0 and σ_1 are mapped to the inner and outer boundary circles of A respectively.

The construction of f begins with finding a solution u to the Dirichlet problem $\Delta u = 0$ on $\Sigma \setminus (\sigma_0 \cup \sigma_1)$, with boundary conditions $u = 0$ on σ_0 , and $u = 1$ on σ_1 . Here Δ is the Laplace-Beltrami operator on the surface Σ (see [12]).

We then find a smooth curve C on Σ which runs from σ_0 to σ_1 such that u is strictly increasing along C . This curve defines a cut on Σ , and the cut surface $\Sigma \setminus C$ is conformally equivalent to a rectangle in the plane.

We next compute the harmonic function v which is conjugate to u by specifying boundary conditions on the cut surface and again solving a Dirichlet problem. The details for this are given in Section 7. The mapping

$u + iv : \Sigma \rightarrow \mathbf{C}$ sends the surface Σ to a rectangle, its exponential $f = e^{u+iv}$ sends Σ to an annulus.

3 Approximation of the Flattening Function

In the previous section, we outlined the analytical procedure for finding the flattening map f . Here we will discuss the finite element method for finding an approximation to this mapping. See [9] for details about this method. Since the procedure given here is crucial for the flattening algorithm we have used in our simulations, we will describe this implementation in some detail.

We assume that Σ is a triangulated surface, and we look for a flattening map f which is continuous on Σ and linear on each triangle. Here, we will concentrate on finding u , the method for its conjugate v being similar.

It is a classical result [12] that the harmonic function u is the minimizer of the Dirichlet functional

$$\begin{aligned} \mathcal{D}(u) &:= \frac{1}{2} \int_{\Sigma} |\nabla u|^2 dS, \\ u|_{\partial\sigma_0} &= 0, \quad u|_{\partial\sigma_1} = 1, \end{aligned}$$

where ∇u is the gradient with respect to the induced metric on Σ .

Let $PL(\Sigma)$ denote the finite dimensional space of piecewise linear functions on Σ . For each vertex $V \in \Sigma$, let ϕ_V be the continuous function such that

$$\begin{aligned} \phi_V(V) &= 1, \\ \phi_V(W) &= 0, \quad W \neq V, \\ \phi_V &\text{ is linear on each triangle.} \end{aligned} \tag{1}$$

Now this set $\{\phi_V\}$ forms a basis for $PL(\Sigma)$, and so any $u \in PL(\Sigma)$ can be written as

$$u = \sum_{V \text{ vertex of } \Sigma} u_V \phi_V.$$

So to approximate the solution to the PDE, we minimize $\mathcal{D}(u)$ over all $u \in PL(\Sigma)$ which satisfy the boundary conditions.

To minimize $\mathcal{D}(u)$, we introduce the matrix

$$D_{VW} = \int \int \nabla \phi_V \cdot \nabla \phi_W dS,$$

for any pair of vertices V, W . It is easy to see that $D_{VW} \neq 0$ if and only if V, W are connected by some edge in the triangulation. One can show that $u = \sum_V u_V \phi_V$ minimizes the Dirichlet functional over $PL(\Sigma)$ with the boundary conditions, if for each vertex $V \in \Sigma \setminus (\sigma_0 \cup \sigma_1)$,

$$\sum_{W \in \Sigma \setminus (\sigma_0 \cup \sigma_1)} D_{VW} u_W = - \sum_{W \in \sigma_1} D_{VW}. \quad (2)$$

3.1 Computing D_{VW}

Assume $V \in \Sigma \setminus (\sigma_0 \cup \sigma_1)$. If $V \neq W$, then the edge VW belongs to two triangles, VWX , and VWY . An essential formula from finite-element theory [9], then says that

$$D_{VW} = -\frac{1}{2} \{ \cot \angle X + \cot \angle Y \},$$

where $\angle X$ is the angle at the vertex X in the triangle VWX , and $\angle Y$ is the angle at the vertex Y in the triangle VWY .

If $V = W$, then one has

$$D_{VV} = - \sum_{W \neq V} D_{VW}.$$

3.2 Summary of Method

We may summarize the finite element procedure for the construction of the flattening map as follows:

- (1) Compute the D_{VW} .
- (2) Solve the linear equation (2) to obtain the piecewise linear harmonic function $u = \sum_V u_V \phi_V$.
- (3) Cut the surface from σ_0 to σ_1 , and compute the boundary values of v by integrating $\frac{\partial u}{\partial n}$ (the derivative of u in the normal direction [12]) around the boundary. Use finite elements to solve for v . To find a cut, one may start at a vertex on σ_0 , and move from each vertex to the adjacent vertex which has the largest value of u associated with it. The maximum principle implies that there is always an adjacent vertex with

a larger value of u than the current vertex. Also, in this way we make a cut which follows closely the gradient of u while remaining on edges of triangles. Thus our flattened image will be roughly rectangular in shape.

4 Inspection and Distortion Removal

In practice, once the colon surface has been flattened into a rectangular shape, it will need to be visually inspected for pathologies. In this section, we present a simple technique by which the entire colon surface can be presented to the viewer as a sequence of images or cine. In addition, this method allows the viewer to examine each surface point without distortion at some time in the cine. Here, we will say a mapping is without distortion at a point if it preserves the intrinsic distance there.

It is well known that a surface can not in general be flattened onto the plane without some distortion somewhere (see [5]). However, it may be possible to achieve a surface flattening which is free of distortion along some curve. A simple example of this is the familiar Mercator projection of the earth, in which the equator appears without distortion. See [11] for a nice discussion of the classical geographic projections and their application to virtual colonoscopy. In our case, the distortion free curve will be a level set of the harmonic function u described above (essentially a loop around the tubular colon surface), and will correspond to the vertical line through the center of a frame in the cine.

Specifically, suppose we have conformally flattened the colon surface onto a rectangle $R = [0, u_{max}] \times [-\pi, \pi]$. Let F be the inverse of this mapping, and let $\phi^2 = \phi^2(u, v)$ be the amount by which F scales a small area near (u, v) , i.e. let $\phi > 0$ be the ‘‘conformal factor’’ for F . Fix $w > 0$, and for each $u_0 \in [0, u_{max}]$ define a subset $R_0 = ([u_0 - w, u_0 + w] \times [-\pi, \pi]) \cap R$ which will correspond to the contents of a cine frame. We define a mapping

$$(\hat{u}, \hat{v}) = G(u, v) = \left(\int_{u_0}^u \phi(\mu, v) d\mu, \int_0^v \phi(u_0, \nu) d\nu \right)$$

from R_0 to \mathbf{C} which has differential

$$dG(u, v) = \begin{pmatrix} \hat{u}_u & \hat{u}_v \\ \hat{v}_u & \hat{v}_v \end{pmatrix} = \begin{pmatrix} \phi(u, v) & \int_{u_0}^u \phi_v(\mu, v) d\mu \\ 0 & \phi(u_0, v) \end{pmatrix}$$

and in particular $dG(u_0, v) = \phi(u_0, v) \times \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. The conformality of the flattening map, together with this value for $dG(u_0, v)$, implies that the composition of the flattening map with G sends the level set loop $\{u = u_0\}$ on the colon surface to the vertical line $\{\hat{u} = 0\}$ in the \hat{u} - \hat{v} plane without distortion. In addition, it follows from the formula for dG that lengths measured in the \hat{u} direction accurately reflect the lengths of corresponding curves on the colon surface.

5 Computer Simulations

We illustrate the algorithm on a data set provided to us by the Surgical Planning Laboratory of Brigham and Women's Hospital. This example consists in a test of flattening the colon surface contained in a $256 \times 256 \times 124$ CT colon image. Four slices from this data set are shown in Figure 1.

First, using the fast segmentation methods of [10, 15] we found the colon surface. Unfortunately, the segmentation algorithm itself does not guarantee that the surface found will be a topological cylinder. In fact, it may contain numerous minute handles which arise because the boundary of the colon, as represented in the data set, may not be sharp. We used a morphological based method by which these handles can be effectively removed and a surface which has the topology of a closed-ended cylinder can be extracted. This is done in such a way that the large-scale geometry of the surface is not adversely affected. We then created a triangularization of this surface using the Visualization Toolkit [14]. Next, the triangularized surface was smoothed slightly to reduce the effects of aliasing. This was done by using the flow according to mean curvature. This also allowed us to obtain a measure of the convexity and concavity of points on the colon surface by considering the mean curvature vector. The triangles making up the two ends of the tubular surface were then removed to produce an open-ended tube with two boundary curves.

Next, as described in the Section 3, we made a cut along the colon surface from a point on one of the bounding curves to a point on the other bounding curve, and then conformally flattened the resulting cut surface onto a nearly rectangular region of the plane. We colored points of the colon surface according to mean curvature, and then applied these same colors to

the flattened image in the plane. This gave an elegant way to visualize the structure of the colon which is a twisted, undulating tube.

We should note that the whole procedure described above from segmenting the colon to flattening took about 12 minutes on a Sun Ultrasparc 10 Workstation.

In Figure 2, we show three different views of the segmented colon surface and its flattened representation. A small circled region visible at the bottom of the flattened surface, and the corresponding region of the original surface, is shown in detail in Figure 3. In Figure 4, we show an interior “fly-through” view of the colon image, and its flattened representation. Notice how much easier it is to get a view of the whole interior region in this manner.

Next, we used the formulas in Section 4 to create a distortion correcting cine of a scan of the colon surface. To make numeric integration fast and simple, we interpolated the function ϕ (defined on the rectangular representation of the colon in the $u-v$ plane) onto an equally spaced grid. For each particular value u_0 of u , increasing from 0 to u_{max} on this grid, we mapped the corresponding sub-rectangle R_0 to the $\hat{u}-\hat{v}$ plane as described in Section 4. We found that this could easily be done in real time. Four frames from the resulting cine can be seen in Figure 5. For comparison, we show an exterior view of the portion of the original colon surface corresponding to each frame. As the cine progresses, the vertical line through the center of the frame is a distortion correcting representation of a loop on the colon surface. These loops sweep continuously over the colon surface from end to end, and thus every surface point is presented to the viewer in some frame without distortion. Further, the mapping to the $\hat{u}-\hat{v}$ plane is a continuous function of time, and so there is no jumping between frames. The fact that distances in the \hat{u} direction are the same as corresponding distances on the surface seems to help in keeping distortion to an acceptable level even off the vertical center line.

Although these images show only mean curvature, we believe that other geometric quantities may also be useful in the visualization of the flattened surface. Polyps, for example, should have relatively high Gaussian curvature as compared to the flatter surrounding colon surface. It is possible that thickness of the colon wall may also be used to distinguish these pathologies.

6 Conclusions

In this note, we outlined a high-level procedure based on conformal geometry and harmonic analysis for the construction of a flattening map of a colon surface derived from volumetric CT data. Further, we presented a numerical algorithm which finds this map based on finite elements. Finally, we demonstrated formulas which can be used real time in a cine to provide a distortion free view of every point on the colon surface.

Additional details on the underlying mathematics, numerics, and an application to 3D brain imagery may be found in [1].

Deformations of highly undulated surfaces have many uses for both military and medical imagery, as well as computer graphics in the area of texture mappings. In this paper, we have proposed a novel solution to the problem of surface deformation in a manner in which angles are locally preserved.

We have also been considering area-preserving flattening maps of minimal distortion, that is, to try to optimize the trade-off of exact area-preservation with minimal angle deviation in our flattening maps. It is mathematically impossible to have both the area and angles preserved everywhere in a diffeomorphism between surfaces unless the two surfaces have the same Gaussian curvature; see [5] for all the relevant results and definitions. Thus the type of trade-off mentioned above is probably the best that one can do. For the mathematical details see [2]. We will be applying this latter methodology to both brain and colon imagery in our future work.

7 Mathematical Appendix

We use the notation of Section 2. We will fill in the details of the high-level algorithm how to construct the flattening map $f : \Sigma \rightarrow \mathbf{C}$.

- (1) Let u be the solution of the Dirichlet problem

$$\begin{aligned}\Delta u &= 0 \text{ on } \Sigma \setminus (\sigma_0 \cup \sigma_1), \\ u &= 0 \text{ on } \sigma_0, \\ u &= 1 \text{ on } \sigma_1.\end{aligned}\tag{3}$$

Here Δ represents the Laplace-Beltrami operator on the surface Σ . This second order partial differential operator is intrinsic to the surface

and is a generalization of the standard Laplacian in the plane. From the standard theory of partial differential equations [12], u exists and is unique.

- (2) Let C be a smooth curve on Σ which runs from σ_0 to σ_1 such that u is strictly increasing along C . The maximum principle implies the existence of such a curve. This curve defines a cut on Σ , and the cut surface $\Sigma \setminus C$ is conformally equivalent to a rectangle in the plane.

Let B be the oriented boundary of this cut surface, i.e. let B run around σ_0 , then along C to σ_1 , around σ_1 , then along C again in the opposite direction to σ_0 . The closed path B must run around σ_0 and σ_1 in a fashion consistent with an orientation given on the surface Σ .

We want to compute the harmonic function v which is conjugate to u by specifying boundary conditions on the cut surface and again solving a Dirichlet problem. Let $\frac{\partial}{\partial s}$ and $\frac{\partial}{\partial n}$ denote tangential and normal derivatives respectively. By the Cauchy-Riemann equations, one has

$$\frac{\partial v}{\partial s} = \frac{\partial u}{\partial n}$$

on B and so the boundary values of v may be found by integration along B ,

$$v(\zeta) = \oint_{\zeta_0}^{\zeta} \frac{\partial v}{\partial s} ds = \oint_{\zeta_0}^{\zeta} \frac{\partial u}{\partial n} ds,$$

where here s is the arc-length along B . Since u is harmonic, the divergence theorem guarantees that $\oint_B \frac{\partial u}{\partial n} ds = 0$, and so v is smooth (periodic) on B .

Note that if we choose C so that it follows the gradient of u from σ_0 to σ_1 , then we would have $\frac{\partial u}{\partial n} = 0$ along C , and so v would be constant on B along the parts corresponding to C .

- (3) The component v of the rectangular map $f_1 = u + iv$ is then found by solving the Dirichlet problem using the calculated boundary values for v . If C is chosen to follow along the gradient of u , then the image of Σ in the u - v plane under f_1 will be a rectangle of width 1 and some height h . For other curves C , along which u is monotone, the image will be a region in the u - v plane between the lines $u = 0$, $u = 1$, the graph of

a function $g = g(u)$, and a copy of the graph of g shifted vertically by a constant, i.e. $g(u) + h$. The fact that the distance across the region vertically is constant is again a consequence of the divergence theorem.

We may now dilate the flattened rectangular image of Σ under f_1 homothetically, and so we may assume that $h = 2\pi$. We may then map the surface to a true cylinder of radius 1 by simply “rolling up” the flattened image. We may also map it to an annulus via $f = e^{f_1}$, effectively reconnecting the image of the surface smoothly across the cut. We may then map it to the sphere using stereographic projection, if desired. In practice, we have found the rectangular mapping to be the most natural representation of the flattened colon. The existence of the cut in the rectangular mapping does not present a problem for visualization; the constant height of the rectangular image allows us to extend it periodically above and below the cut.

Remark: The annular map f is unique up to a dilation, rotation and translation. That is, any other conformal $\hat{f} : \Sigma \rightarrow \mathbf{C}$ which maps Σ to an annulus and σ_1 to its outer boundary is given by

$$\hat{f} = \alpha f + \beta, \tag{4}$$

for some constants $\alpha, \beta \in \mathbf{C}$.

References

- [1] S. Angenent, S. Haker, A. Tannenbaum, R. Kikinis, “Laplace-Beltrami operator and brain surface flattening,” *IEEE Trans. Medical Imaging*, October 1999.
- [2] S. Angenent, S. Haker, A. Tannenbaum, R. Kikinis, “On area preserving mappings of minimal distortion,” in *System Theory: Modeling, Analysis, and Control*, edited by T. Djaferis and I. Schick, Kluwer, Holland, 1999, pages 275-287.
- [3] H. Farkas and I. Kra, *Riemann Surfaces*, Springer-Verlag, New York 1991.

- [4] A. Dale and M. Sereno, "Improved localization of cortical activity by combining EEG and MEG with MRI cortical surface reconstruction: a linear approach," *Journal of Cognitive Neuroscience* **5** (1993), pp. 162-176.
- [5] M. P. Do Carmo, *Riemannian Geometry*, Prentice-Hall, Inc. New Jersey, 1992.
- [6] H. Drury, D. van Essen, C. Anderson, C. Lee, T. Coogan, and J. Lewis, "Computerized mappings of the cerebral cortex: a multiresolution flattening method and a surface-based coordinate system," *Journal of Cognitive Neuroscience* **8** (1996), pp. 1-28.
- [7] A. Hara, C. Johnson, J. Reed, "Detection of colorectal polyps by computed tomographic colography: feasibility of a novel technique," *Gastroenterology* **110** (1996), pp. 284-290.
- [8] A. Hara, C. Johnson, J. Reed, R. Ehman, D. Ilstrup, "Colorectal polyp detection with CT colography: two- versus three-dimensional techniques," *Radiology* **200** (1996), pp. 49-54.
- [9] T. Hughes, *The Finite Element Method*, Prentice-Hall, New Jersey, 1987.
- [10] S. Kichenasamy, P. Olver, A. Tannenbaum, A. Yezzi, "Conformal curvature flows: from phase transitions to active contours," *Archive Rational Mechanics and Analysis* **134** (1996), pp. 275-301.
- [11] D. Paik, C. Beaulieu, R. Jeffrey, C. Karadi, and S. Napel, "Visualization modes for CT colonography using cylindrical and planar map projections," Technical Report, Department of Radiology, Stanford University School of Medicine, Stanford, CA, 1999.
- [12] J. Rauch, *Partial Differential Equations*, Springer-Verlag, New York 1991.
- [13] G. Rubin, S. Napel, and A. Leung, "Volumetric analysis of volumetric data: achieving a paradigm shift," *Radiology* **200** (1996), pp. 312-317.
- [14] W. Schroeder, H. Martin, B. Lorensen, *The Visualization Toolkit*, Prentice-Hall, New Jersey, 1996.

- [15] K. Siddiqi, A. Tannenbaum, and S. Zucker, "Area and length minimizing flows for image segmentation," *IEEE Trans. Image Processing* **7** (1998), pp. 433-444.
- [16] D. Vining, "Virtual endoscopy: is it reality?," *Radiology* **200** (1996), pp. 30-31.
- [17] B. Wandell, S. Engel, and H. Hel-Or, "Creating images of the flattened cortical sheet," *Invest. Opth. and Vis. Sci.* **36** (S612), 1996.
- [18] G. Wang, E. McFarland, B. Brown, and M. Vannier, "GI tract unraveling with curved cross sections," *IEEE Trans. Med. Imaging* **17** (1998), pp. 318-322.

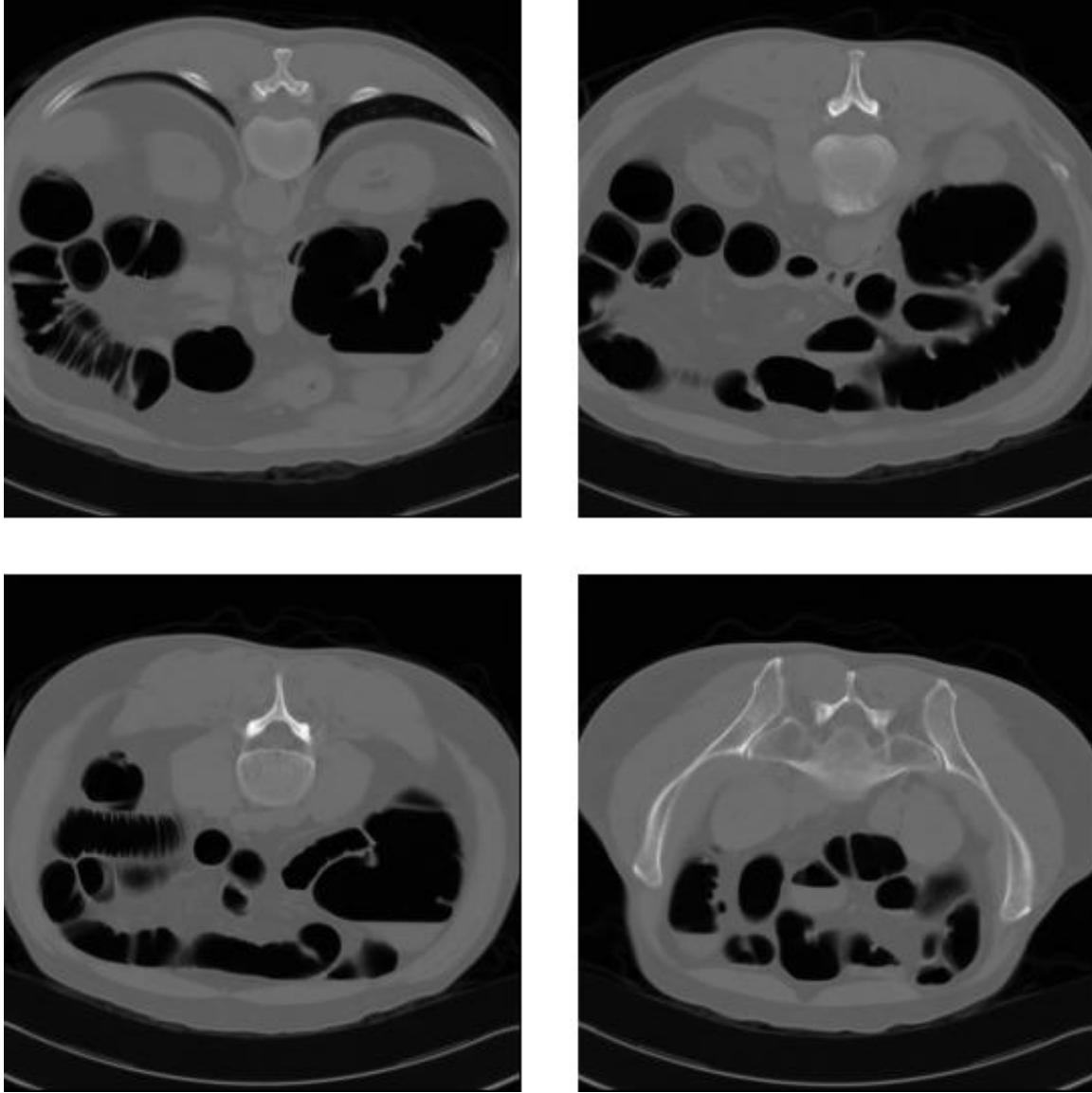


Figure 1: Four Slices of CT Colon Data



Figure 2: Three Views of Segmented Colon and Flattened Representation

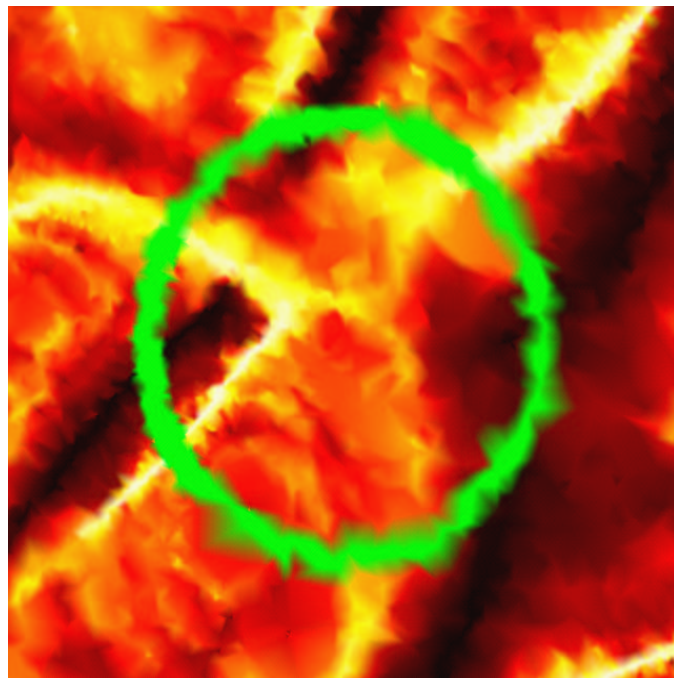
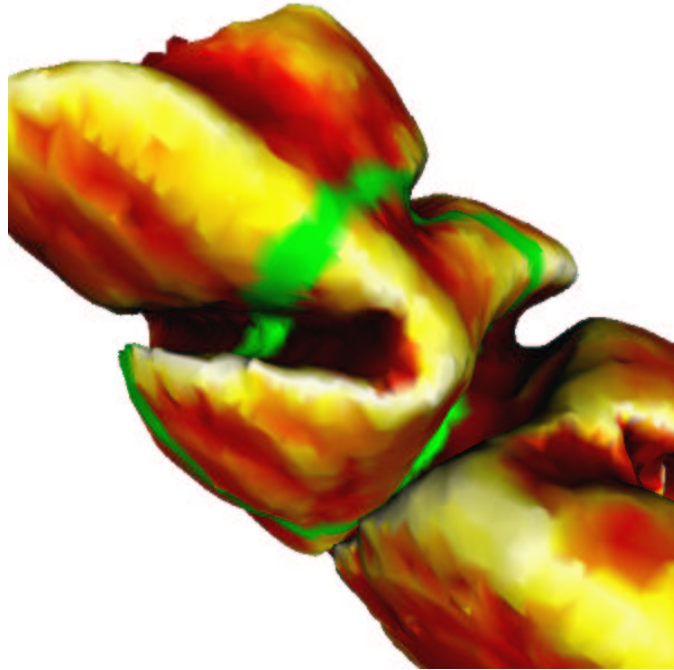


Figure 3: Region of Interest in Segmented Data and Flattened Representation

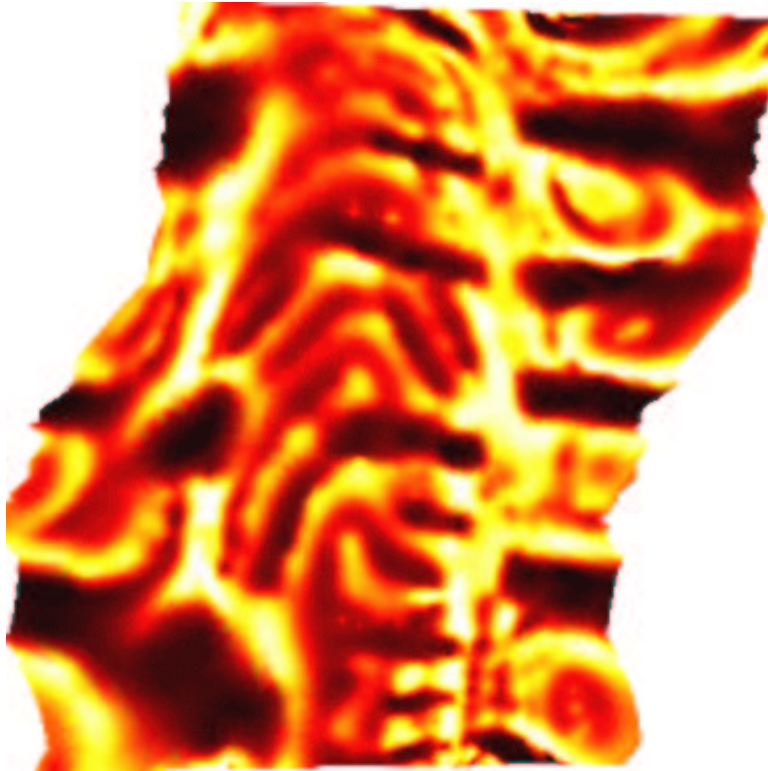
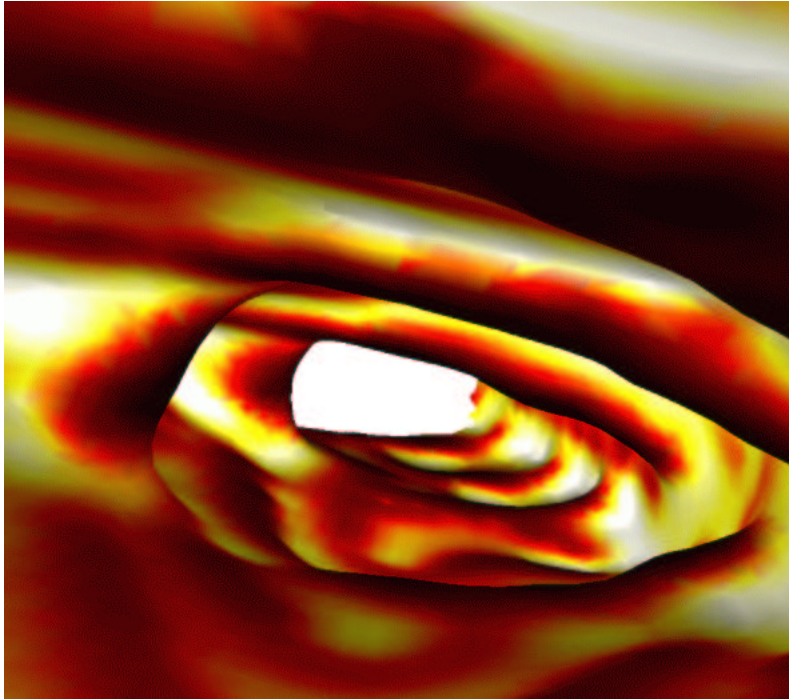


Figure 4: “Fly-Through” View and Flattened Representation

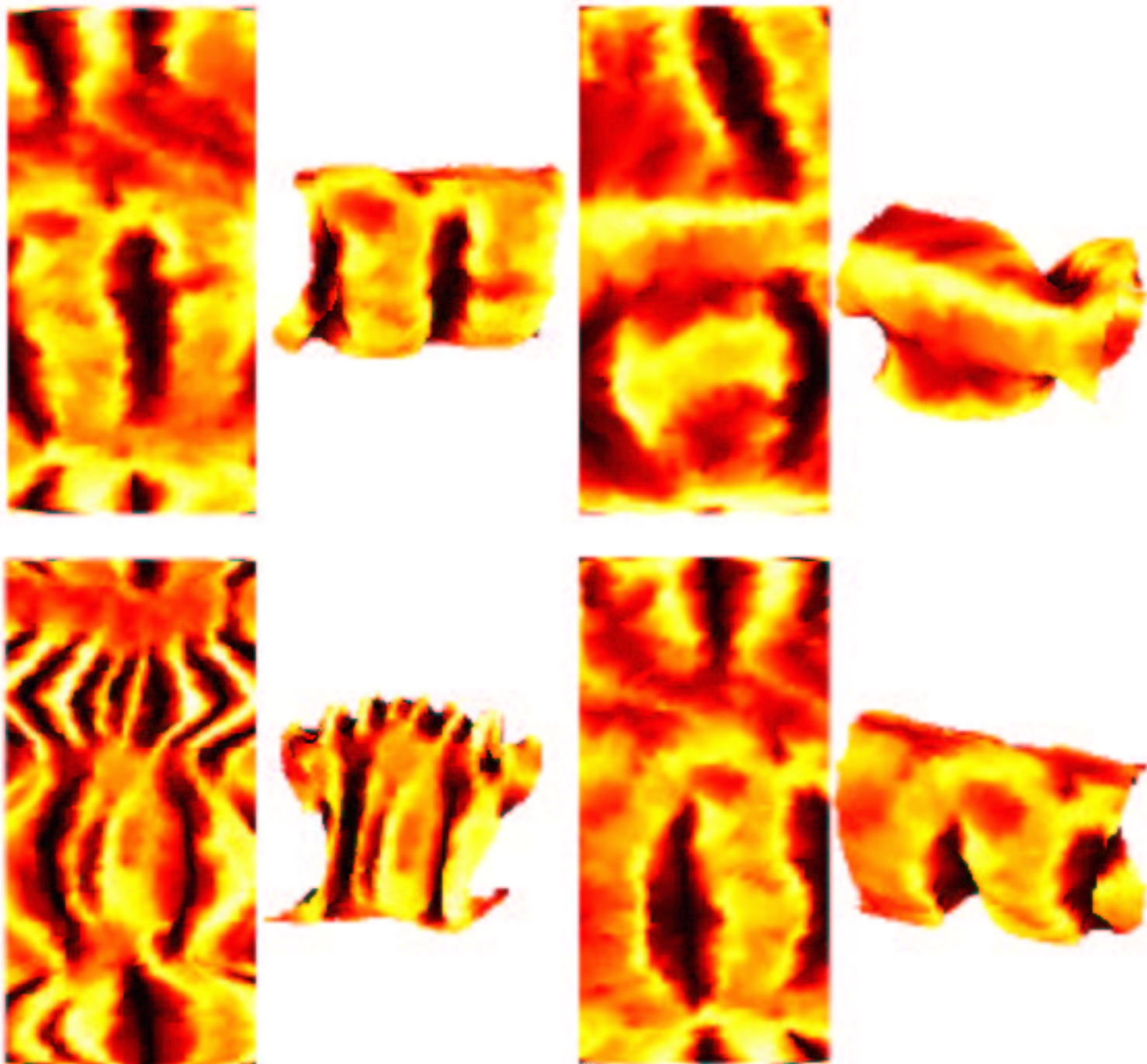


Figure 5: Frames From Distortion-Correcting Cine with Original Surface