Curriculum Vitae

Florian BERTRAND

French citizenship.
Born on 02/23/1980 in Gonesse (France).

1 Curriculum Vitae

Van Vleck Hall
3480 Lincoln Dr, Madison
Madison, 53706, WI, USA
Office 421
Phone : 608 263 6395
E-mail : bertrand@math.wisc.edu
Web-page : http ://www.math.wisc.edu/∼bertrand/

I am a Van Vleck Assistant Professor at the University of Wisconsin, Madison, USA.

Education

2004–2007 **PhD student and teaching assistant** at the LATP at the University of Provence (Marseille, France). I defended my PhD "Analyse locale dans les variétés presque complexes" on the 7th of December 2007 in front of the following jury :

M. Bernard Coupet	University of Provence	Examiner
M. Hervé Gaussier	University of Provence	Supervisor
M. Sergey Ivashkovich	University of Lille I	Reader
Mme. Christine Laurent	University of Grenoble I	Examiner
M. Jean-Jacques Loeb	University of Angers	Reader

2003–2004 **Master, second year** in Mathematics at the University of Provence (Marseille, France).

2002–2003 **Agrégation in Mathematics** (highest-level french national competitive exam for high-school teacher recruitment).

2001–2002 **Master, first year** in Mathematics at the University of Provence (Marseille, France).

1998–2001 **Bachelor's degree** in Mathematics at the University of Provence (Marseille, France).

Professionnal evolution

2009–2011 **Assistant Professor** at the University of Wisconsin, Madison, USA, from January 2009.

Fall 2008 **Fields Postdoctoral Fellow** at the Fields Institute, Toronto, Canada.

2007–2008 **ATER** at the University of Provence.

2004–2007 **Teaching assistant** at the Université of Provence.

2 Teaching activities

Spring 2009

- Math. 320, Differential equation and linear algebra.
- Math. 521, Advanced Calculus.

2007–2008

- Topology, Bachelor's Degree (51 hours).
- Function series, Bachelor's Degree (51 hours).
- Analysis, Bachelor's Degree (24 hours).
- Measure Theory, Bachelor's Degree (36 hours).
- Complex analysis, Bachelor's Degree (24 hours).

2006–2007

- Topology, Bachelor's Degree (64 hours).

2005–2006

- Linear algebra, Bachelor's Degree (64 hours).

2004–2005

- Probability and Statistics, Bachelor's Degree (64 hours).

3 Publications

- Almost complex structures on the cotangent bundle, Published in Complex Variables and Elliptic Equations, Volume 52, Issue 8, August 2007, 741-754.
- Sharp estimates of the Kobayashi pseudometric and Gromov hyperbolicity, Journal of Mathematical Analysis and Applications, Volume 345, Issue 2, September 2008, 825-844.
- Pseudoconvex regions of finite D'Angelo type in four dimensional almost complex manifolds, to be published in Mathematische Zeitschrift.

4 Research activities

Presentations

- Estimates of the Kobayashi pseudometric in almost complexe manifolds,
  - January 2009, analysis seminar, UW-Madison (US).
  - September 2008, workshop on hyperbolic geometry, Toronto (Canada).
  - April 2008, analysis and geometry seminar, Paris (France).
  - March 2008, French-Korean workshop of complex analysis and geometry, Marseille (France).
  - March 2008, complex analysis, geometry and dynamic seminar, Toulouse (France).
- Sharp estimates of the Kobayashi metric and Gromov hyperbolicity
  - January 2008, analysis seminar d'analyse, Poitiers (France).
  - January 2008, analysis seminar, Grenoble (France).
  - October 2007, journées complexes du sud, Rosas (Spain).
- Pseudoconvex domains of finite type in almost complex manifolds,
  - May 2007, geometric analysis seminar, Lille (France).
  - April 2007, complex analysis workshop "special Week CMI de Provence - BK21 Codimaro of Postech", Pohang (South Korea).
  - December 2006, analysis and geometry seminar, Marseille (France).
- The geometry of the Poincaré hyperbolic disc,
  - April 2007, PhD students seminar, Marseille (France).
- Almost complex structures on the cotangent bundle
  - July 2005, summer school on real PDE's for complex and CR geometry, Trento (Italy).

Conferences participations

- Mars 2008 : French-Korean workshop of complex analysis and geometry, Marseille CIRM (France).
- Octobre 2007 : Journées complexes du sud, Rosas (Espagne).
- Septembre 2007 : Rencontres d'analyse et géométrie complexes, Marseille CIRM (France).
- Avril 2007 : Workshop "Special Week CMI de Provence - BK21 Codimaro of Postech", Pohang (South Korea).
- Novembre 2006 : Journées complexes du sud, Les Rasses (Switzerland).
- Avril 2006 : Journées complexes du sud, Marseille CIRM (France).
- Septembre 2005 : Conference on Complex Analysis in honour of Henri Skoda, Paris IHP (France).
- Juillet 2005 : Summer school on real PDE's for complex and CR geometry, Trento (Italy).
- Juin 2005 : Complexe analysis and geometry conference, Marseille CIRM (France).
- Juillet 2004 : Summer school on complex analysis and geometry, Mahdia (Tunisie).

Responsabilities

- Reviewer for Mathematical Reviews (AMS).

5 Description of my research works

My main research interest is the almost complex geometry. More precisely, I work on local analysis in almost complex manifolds : pseudoholomorphic maps, Kobayashi pseudometric, construction of plurisubharmonic functions.

Almost complex structures on the cotangent bundle.

Several lifts of an almost complex structure on a base manifold are constructed on the cotangent bundle. These are essentially due to I.Sato in [16] and S.Ishihara and K.Yano in [9]. I.Sato defined a lift of the ambient structure as a correction of the formal complete lift and S.Ishihara and K.Yano introduced the horizontal lift obtained via a symmetric connection. The aim of this work is to unify and to generalize these lifts by introducing a more natural almost complex lift called the generalized horizontal lift.

It turns out that my construction depends on the introduction of some connection : I study the dependence of the lift on it. My main result states that the structure defined by I.Sato and the horizontal lift are special cases of my general construction, obtained by particular choices of connections. I establish some geometric properties of this general lift and I characterize generically the structure constructed by I.Sato by the holomorphicity of the lift of a given diffeomorphism on the bases and by the holomorphicity of the complex fiberwise multiplication.

Finally, I study the compatibility between lifted almost complex structures and symplectic forms on the cotangent bundle. The conormal bundle of a strictly pseudoconvex hypersurface is a totally real maximal submanifold in the cotangent bundle endowed with the structure defined by I.Sato. One can search for a symplectic proof of this, since every Lagrangian submanifold in a symplectic manifold is totally real for almost complex structures compatible with the symplectic form. I prove that for every almost complex manifold and every symplectic form on the cotangent bundle compatible with the generalized horizontal lift, the conormal bundle of a strictly pseudoconvex hypersurface is not Lagrangian. This illustrates the singular fact that to study local complex (or almost complex) geometry, we naturally use structures which are not compatible with the canonical symplectic form of the cotangent bundle.

Pseudoconvex domains of finite D'Angelo type in almost complex manifolds of dimension four.

In this paper, I study the behaviour of the Kobayashi pseudometric of a J-pseudoconvex region $D = \{\rho < 0\}$, where ρ is a $\mathcal{C}^2$ defining function of D, J-plurisubharmonic on a neighborhood of $\overline{D}$, of finite D'Angelo type in an almost complex manifold (M, J) of dimension four.

My main result is a generalization of a result of J.E.Fornaess and N.Sibony [6] : I construct a local peak J-plurisubharmonic function at any boundary point of D. This allows to localize pseudoholomorphic discs and to obtain lower estimates of the Kobayashi pseudometric which provide the local Kobayashi hyperbolicity of J-pseudoconvex regions of D'Angelo type $2m$. As an application I prove the $1/2m$-Hölder extension of pseudolomorphic diffeomorphisms up to the boundary.

In order to obtain precise lower estimates of the Kobayashi pseudometric similar to those given in complex manifolds by D.Catlin [3] (see also the paper of F.Berteloot [2]), I consider a natural scaling method. However this reveals the fact that for a domain of finite D'Angelo type greater than four, the sequence of almost complex structures obtained by any polynomial scaling process does not converge generically to the standard structure. This may be related to the fact that finite D'Angelo type is based on purely complex considerations, as the boundary behaviour of the Cauchy-Riemann equations. Hence I provide precise lower estimates of the Kobayashi pseudometric for a pseudoconvex region of finite D'Angelo type four. I point out that the approach I use, based on some renormalization principle of pseudoholomorphic discs, also gives a different proof of precise lower estimates obtained by H.Gaussier and A.Sukhov in [7] for strictly J-pseudoconvex domains in arbitrary dimension. I obtain the (local) complete hyperbolicity of J-pseudoconvex regions of D'Angelo type less than or equal to four and I give a Wong-Rosay theorem for regions with a noncompact automorphisms group.

Finally, in order to obtain precise estimates near a point of arbitrary finite D'Angelo type, I am interested in the nontangential behaviour of the Kobayashi pseudometric.

Gromov hyperbolicity of strictly pseudoconvex domains in almost complex manifolds of dimension four.

In this work, I give sharp estimates of the Kobayashi pseudometric of any relatively compact strictly J-pseudoconvex smooth domain in a four dimensional almost complex manifold (M, J). My proof is inspired by a result by D.Ma [13]. However the proof he gives is based on some purely complex analysis argument as the existence of peak holomorphic function. I bypass this obstruction by considering a quantitative approach using a well chosen family of polydiscs. Notice that this also gives a different way to obtain estimates in [13] in complex manifolds without using any complex analysis tools.

In the complex Euclidean space, Z.M.Balogh and M.Bonk [1] proved the Gromov hyperbolicity of strictly pseudoconvex domains. Their proof is based on sharp estimates of the Kobayashi pseudometric obtained by D.Ma [13], and on some sub-Riemannian geometry. In particular their proof remains valid in the almost complex setting and, consequently, any relatively compact strictly J-pseudoconvex smooth domain D in an almost complex manifold (M, J) of dimension four is Gromov hyperbolic with respect to the Kobayashi distance.

Références

[1] Balogh,Z. M., Bonk,M. *Gromov hyperbolicity and the Kobayashi metric on strictly pseudoconvex domains*, Comment. Math. Helv. **75** (2000), 504-533.

[2] Berteloot,F. *Principe de Bloch et estimations de la métrique de Kobayashi dans les domaines de* $\mathbb{C}^2$, J. Geom. Anal. **13** (2003), 29-37.

[3] Catlin, D. *Estimates of invariant metrics on pseudoconvex domains if dimension two*, Math. Z. **200** (1989), 429-466.

[4] Coupet,B., Gaussier,H., Sukhov,A. *Riemann maps in almost complex manifolds*, Ann. Sc. Norm. Super. Pisa Cl. Sci. (5) **2** (2003), 761-785.

[5] Debalme,R., Ivashkovich,S. *Complete hyperbolic neighborhoods in almost complex surfaces*, Int. J. Math. **12** (2001), 211-221.

[6] Fornaess,J.E., Sibony,N. *Construction of p.s.h. functions on weakly pseudoconvex domains*, Duke Math. J. **58** (1989), 633-655.

[7] Gaussier,H., Sukhov,A. *Estimates of the Kobayashi metric on almost complex manifolds*, Bull. Soc. Math. France **133** (2005), 259-273.

[8] Gromov,M. *Pseudoholomorphic curves in symplectic manifolds*, Invent. Math. **82** (1985), 307-347.

[9] Ishihara,S., Yano,K. *Tangent and cotangent bundles*, Marcel Dekker NY (1973).

[10] Ivashkovich,S., Rosay,J.-P. *Schwarz-type lemmas for solutions of $\overline{\partial}$-inequalities and complete hyperbolicity of almost complex manifolds*, Ann. Inst. Fourier **54** (2004), 2387-2435.

[11] Ivashkovich,S., Pinchuk,S., Rosay,J.-P. *Upper semi-continuity of the Kobayashi-Royden pseudo-norm, a counterexample for Hölderian almost complex structures*, Ark. Mat. **43** (2005), 395-401.

[12] Lempert,L. *La métrique de Kobayashi et la représentation des domaines sur la boule*, Bull. Soc. Math. France **109** (1981), 427-474.

[13] Ma,D. *Sharp estimates of the Kobayashi metric near strongly pseudoconvex points*, The Madison Symposium on Complex Analysis (Madison, WI, 1991), Contemp. Math. **137**, Amer. Math. Soc., Providence, RI, 1992, 329-338.

[14] Nijenhuis,A., Woolf, W. *Some integration problems in almost-complex and complex manifolds*, Ann. Math. **77** (1963), 429-484.

[15] Rosay,J.-P. *J-holomorphic submanifolds are pluripolar*, Math. Z. **253** (2006), 659-665.

[16] Sato,I. *Almost analytic vector fields in almost complex manifolds*, Tohoku Math. J. **17** (1965), 185-199.