

MATH 580/780I MIDTERM 1, FALL 2006

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For full credit you must explain your reasoning. Each question is worth an equal amount.

1. (a) For $n \geq 1$, show that $1^2 + 3^2 + 5^2 + \dots + (2n - 1)^2 = \binom{2n+1}{3}$
(b) Clearly state the binomial theorem.

2. (a) Show that when n is odd, $n^2 - 1$ is a multiple of 8.
(b) Show that if n is a prime greater than 3, then $n^2 - 1$ is a multiple of 24.

3. (a) Clearly state the fundamental theorem of arithmetic.
(b) If a and b are integers, does $a^2 \mid b^3$ imply $a \mid b$? Prove or disprove.

4. (a) Use the Euclidean Algorithm to obtain integers x and y satisfying $\gcd(48, 60) = 48x + 60y$.
(b) What is the $\text{lcm}(48, 60)$?

5. A group of 74 English schoolchildren are to be divided up into teams for soccer (consisting of 11 children each) and for rugby union (consisting of 15 students each). We wish to do this exactly, with no child left out.
(a) Write down a linear Diophantine equation for the problem.
(b) Find one solution to the Diophantine equation (I strongly suggest you do NOT use the Euclidean algorithm here).
(c) Write down the general solution to the Diophantine equation and hence show that there is a unique way of dividing the children up.