

MATH 580/780I MIDTERM 2, FALL 2006

NIGEL BOSTON

For full credit you must explain your reasoning. Each question is worth an equal amount.

1. Let n be of the form $x^4 + 1$, where x is an integer.
 - (a) Quote a theorem from class that implies that n is not divisible by any prime of the form $4k + 3$.
 - (b) Show that n is not divisible by 5 and is not divisible by 13.

2.
 - (a) Find all solutions to the linear congruence $10x \equiv 37 \pmod{25}$.
 - (b) Find all integers x that satisfy both $x \equiv 5 \pmod{6}$ and $x \equiv 3 \pmod{11}$.

3.
 - (a) Find the last two digits of $10!$.
 - (b) Determine whether or not 123456789 is divisible by 99. [Hint: $99 = 9 \times 11$.]

4. Bertrand's conjecture (which is a theorem) states that given any $n \geq 2$, there is a prime between n and $2n$.
 - (a) Show that given any $n \geq 2$, there exists a prime p such that $p \leq n < 2p$.
 - (b) Show that for any $n \geq 2$, $n!$ is not a perfect square.

5.
 - (a) State Fermat's little theorem and define what a pseudoprime is.
 - (b) Show that if p is prime and $n = 2^p - 1$ is composite, then n is a pseudoprime (to base 2).