6. Call a smooth projective curve $C$ over $\mathbb{F}_q$ maximal if $|C(\mathbb{F}_q)| = q + 1 + 2g\sqrt{q}$ ($g$ being its genus).

(a) Show that there are no maximal curves of positive genus over $\mathbb{F}_2$.
(b) Show that the Hermitian curve $x^{q+1} + y^{q+1} + z^{q+1} = 0$ over $\mathbb{F}_{q^2}$ is maximal.
(c) If a curve is maximal, what are the $\alpha_i$ that appear in the numerator of its zeta function?
(d) If $C$ is a maximal curve over $\mathbb{F}_q$, compute $|C(\mathbb{F}_{q^2})|$. Deduce an upper bound for its genus in terms of $q$. [Remark: maximal curves arise in coding theory and finance applications.]