SUMMATION INVARIANT AND ITS APPLICATIONS TO SHAPE RECOGNITION

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ABSTRACT

A novel summation invariant of curves under transformation group action is proposed. This new invariant is less sensitive to noise than the differential invariant and does not require an analytical expression for the curve as the integral invariant does. We exploit this summation invariant to define a shape descriptor called a semi-local summation invariant and use it as a new feature for shape recognition. Tested on a database of noisy shapes of fishes, it was observed that the summation invariant feature exhibited superior discriminating power than that of wavelet-based invariant features.

1. INTRODUCTION

Invariants for transformation groups play an important role in computer vision. The idea that one can compute functions of images that do not change under various viewing conditions is appealing. It holds potential for many applications. Hence the study of invariants for certain transformation groups (Euclidean, affine and projective) has flourished.

Toward the end of the last millennium, algorithms based on invariants did not meet our expectations. Differential invariants depend on derivatives so are very sensitive to noise [1, 2]. There have been several attempts to decrease sensitivity to noise. To avoid high-order derivatives, a semi-differential invariant was introduced in [3]. Affine invariant multiscale analysis was investigated in [4]. Potentials were used as coordinates to prolong group actions, so that the resulting invariants would depend on integrals rather than derivatives and not be sensitive to noise [5]. Also another type of integral invariant was formulated by integrating with respect to affine quasi-invariant arc-length [6].

In this paper, we introduce a general method to generate invariants that are weighted summations of discrete data, as analogues to integral ones. We use the summation invariants to measure the similarity between shapes and illustrate the potential of our method in real world applications. The rest of this paper is organized as follows. Section 2 describes the summation invariant. In section 3, we use the summation invariant to define a novel shape descriptor, which is called the semi-local summation invariant. In section 4, we apply the proposed method to the problem of fish recognition. Finally, section 5 summarizes the contribution and provides an overview of future directions.

2. SUMMATION INVARIANT

The transformation groups acting on $\mathbb{R}^2$, such as Euclidean, affine and projective groups are of particular importance in planar shape recognition. In this section, we describe a systematic method to find invariant functions for certain transformation groups.

2.1. Extending group action to potentials

The boundary of an object is extracted and parameterized as $x[n]$ and $y[n]$. Consider a transformation group G of dimension r acting on $\mathbb{R}^2$ defined by

$$g \circ (x[n], y[n]) = (\overline{x}[n], \overline{y}[n]), g \in G$$

We prolong the group action to the jet space J^n consisting of potentials up to the n-th order. The definition of potential and jet space are shown below.

Definition 1. The potential $P^{i,j}$ of order k is given by

$$P^{i,j} = \sum_{n=0}^{N-1} x^i[n] \cdot y^j[n] \text{, where } i + j = k$$

Definition 2. The jet space J^n is the Euclidean space with coordinates

$$(x[0], y[0], x[N-1], y[N-1], P_{(n)})$$

where $P_{(n)}$ consist of potentials up to *n*-th order.

For example,

$$J^1 = (x[0], y[0], x[N-1], y[N-1], \sum_{n=0}^{N-1} x[n], \sum_{n=0}^{N-1} y[n])$$

Then, we can find an invariant function of the transformation group G by solving the normalization equations [7].

2.2. An example: invariant of affine transformation

In this section, we use affine transformation as an example to illustrate the proposed method. Consider the affine transformation group acting on $\mathbb{R}^2$ given by,

$$g \circ (x, y) = (ax + by + c, dx + ey + f)$$

, where $\det(\begin{bmatrix} a & b \\ c & d \end{bmatrix}) \neq 0$

After applying the group action, $P^{1,0}$ is given by

$$\overline{P}^{1,0} = \sum_{n=0}^{N-1} ax[n] + by[n] + c = aP^{1,0} + bP^{0,1} + cN$$

Similarly,

$$\overline{P}^{0,1} = \sum_{n=0}^{N-1} dx[n] + ey[n] + f = dP^{1,0} + eP^{0,1} + fN$$

We can solve for {a, b, c, d, e, f} by setting

$$(\overline{x}[0], \overline{y}[0], \overline{x}[N-1], \overline{y}[N-1], \overline{P}^{1,0}, \overline{P}^{0,1}) \\ = (0, 0, 1, 1, 0, 0)$$

Then, Substituting {a, b, c, d, e, f} into $\overline{P}^{2,0}$ gives us the invariant function

$$a = \begin{pmatrix} P^{2,0}(Ny[0] - P^{0,1})^2 + P^{0,2}(Nx[0] - P^{1,0})^2 \\ -2P^{1,1}(Nx[0] - P^{1,0})(Ny[0] - P^{0,1}) \\ -N(P^{1,0}y[0] - P^{0,1}x[0])^2 \end{pmatrix} \\ / \begin{pmatrix} Nx[N-1]y[0] - Nx[0]y[N-1] + \\ P^{1,0}(y[N-1] - y[0]) - P^{0,1}(x[N-1] - x[0]) \end{pmatrix}^2$$

3. SEMI-LOCAL SUMMATION INVARIANTS

Since the summation invariant is a map from $\mathbb{R}^N$ to $\mathbb{R}$, the dimension of the feature vector is one. In some applications, such as recognition of similar objects, it won't give us accurate recognition results. Instead of doing global summation, we define a summation invariant locally to extract local features of the contour and also expand the dimension of the feature vector. It's called a semi-local summation invariant. The definition of the semi-local summation invariant is given by

$$b[m] = \left(M(x_1 y_0 - x_0 y_1) + P_x(y_1 - y_0) - P_y(x_1 - x_0)\right)^2$$

where

$$P_x = \sum_{n=m}^{m+M-1} x[\bmod(n, N)] \qquad P_y = \sum_{n=m}^{m+M-1} y[\bmod(n, N)]$$

$$x_0 = x[m], \quad y_0 = y[m], \quad x_1 = x[\bmod(m + M - 1, N)], \\ y_1 = y[\bmod(m + M - 1, N)].$$

Here, we can use only the denominator of α to define a semi-local summation invariant. Since it's transformed by

$$g \circ \text{denominator}(\alpha) = (ae - bd)^2 \ \text{denominator}(\alpha)$$

for all $g \in A(2)$. The scaling factor will be canceled when similarity is measured by normalized cross-correlation as follows:

$$r = \frac{\sum_{n=0}^{N-1} b_1[n] b_2[n]}{\sqrt{\sum_{n=0}^{N-1} b_1^2[n] \sum_{m=0}^{N-1} b_2^2[m]}}$$

4. APPLICATIONS TO SHAPE RECOGNITION

In this section, we apply semi-local summation invariant to recognize 2D fish contours under affine transformations.

4.1. Fish recognition

We randomly selected 100 distinct fish contours from the SQUID database [8], and re-sampled each 2D contour curve such that the total number of points is 512. Some of these fish contours are shown in Figure 1(a). For each curve, we generate 20 variations by applying affine transformations with randomly generated parameters. In

Figure 1(b), we illustrate the 20 variations of the same fish contour. In all, there are 100 distinct types of fish contours and 20 variations for each type. For each type of fish, its 20 variations are partitioned equally and randomly into five disjoint sets of 4 samples each. Combining the corresponding disjoint sets of each fish type, we have five sets of data, with each consisting of exactly four samples of each of the 100 types of fishes.

Fish recognition is performed by choosing SET 1 as the training set and the others as the test set. We can also perform 4 other recognition experiments by choosing SET 2, SET 3, SET 4 and SET 5 as the training set respectively. Specifically, the same pattern classifier will be applied to 5 different partitions of the available data into training and testing data sets. In the k*th* partition, the k*th* data set will be the training data set, and the remaining four data sets combined will form the testing data set. This way, each data set will be used as the training data set exactly once in the 5 partitions.

The semi-local summation invariant is calculated for each fish contour with M =51 with cyclic extension at the boundary. The feature vector has a dimension of 512.

A nearest neighbor pattern classifier is used for this experiment. For each testing feature vector, its cross-correlation with each of the training feature vectors is computed according to the following formula:

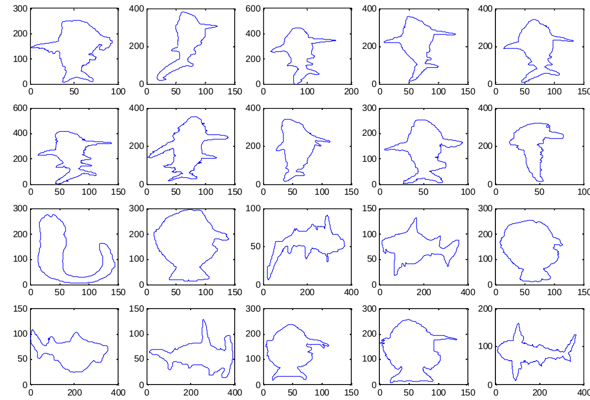
$$r = \frac{\sum_{n=0}^{N-1} h_{training}[n] \cdot h_{test}[n]}{\sqrt{\sum_{n=0}^{N-1} h_{training}^2[n] \sum_{m=0}^{N-1} h_{test}^2[m]}}$$

When the semi-local summation invariant is used for matching fish contours, the total number of mismatches is 177 out of 8000 or 2.21%. We repeat the same procedure but replacing the summation invariant with an integral invariant. The total number of mismatches is 1165/8000 = 14.6%. It is quite clear that the summation invariant based feature performed much better than that of the integral invariant based features.

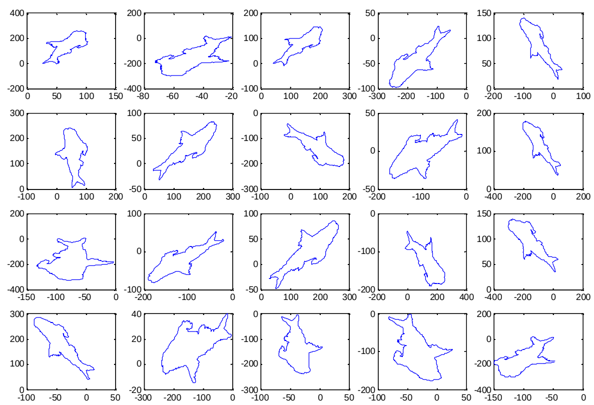
4.2. Sensitivity to noise

Next, the sensitivity of the semi-local summation invariant based feature will be tested experimentally and compared to that of a wavelet invariant feature proposed by Khalil and Bayoumi [9]. For this purpose, we add Gaussian-distributed noise into the fish contour. Again, the database contains 100 distinct types of fish and each type has 20 variations. An example of a fish contour and its noisy version are depicted in Figure 2. Two different noise levels are used: σ = 1, and σ = 2.

It has been shown [9] that the wavelet-based invariant function shows superior discriminating power over other



(a)



(b)

Figure 1. (a) 20 distinct types of fish from the database are shown here. (b) 20 variations of fish contour for the same type of fish.

traditional invariant features such as the moment invariant [10] or Fourier descriptor method [11]. In this work, the wavelet affine invariant function $h_{a,b,c,d,e,f}(t)$ is used. Within the 8 scale-levels we compute $h_{3,4,5,6,7,8}(t)$. During preliminary experiments, it is found that scale levels 1 and 2 are too sensitive to noise and are excluded from the wavelet invariant features.

The results in terms of probability of misclassification are summarized in the following table:

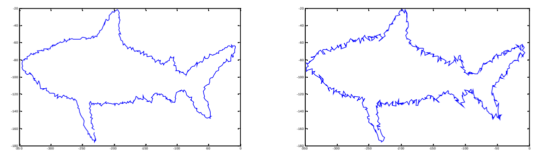
Table 1 Sensitivity comparison using probability of misclassification

	Semi-local	wavelet invariant
σ = 1	367/8000 = 4.6%	2253/8000 = 28.2%
σ = 2	514/8000 = 6.4%	3476/8000 = 43.5%

As we can see, the summation invariant can successfully recognize fish in spite of a high level of noise. Compared with wavelet-based techniques, it also exhibits stronger immunity to noise.

5. CONCLUSION

In this work, we provide a new solution to the equivalence problem of planar contours under transformation group action. A summation invariant for the affine group acting on ¡^{2} is explicitly derived and applied to the problem of shape recognition. A database of marine animals was used to test the proposed method. Compared with some traditional methods, experimental results show that our method has superior discriminating power and better noise immunity. Among possible future directions, the major work is to further improve recognition performance by using higher order summation invariants.



(a) (b)

Figure 2. Fish contours after adding Gaussian distributed noise, (a) $s = 1$**, (b)** $s = 2$**. (**s **denotes the standard deviation of noise)**

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