APPLIED ALGEBRA DAYS ABSTRACTS

Bernd Sturmfels

Abstract: The study of two-dimensional images of three-dimensional scenes is a foundational subject for computer vision, known as multiview geometry. We present recent work with Chris Aholt and Rekha Thomas on the polynomials defining images taken by n cameras. Our varieties are threefolds that vary in a family of dimension 11n-15 when the cameras are moving. We use toric geometry and multigraded Hilbert schemes to characterize degenerations of camera positions.

Ronny Hadani

Abstract: Three dimensional cryo-electron microscopy (3D cryo-EM, for short) is the problem of determining the three dimensional structure of a large molecule from the set of images, taken by an electron microscope, of randomly oriented and positioned identical molecular particles which are frozen in a thin layer of ice. A solution to this problem is of particular interest, since it promises to be an entirely general technique which does not require crystallization or other special preparation stages. Present approaches to the problem fail with particles that are too small, cryo-EM images that are too noisy or at resolutions where the signal-to-noise ratio becomes too small.

In my talk, I will describe a novel algorithm due to Amit Singer and Yoel Shkolnisky, which constitutes a basic step for the solution of the 3D cryo-EM problem, referred to as the intrinsic reconstitution algorithm. The appealing property of this new algorithm is that it exhibits remarkable numerical stability to noise. My main goal is to give a conceptual explanation, based on represen- tation theory, for the admissibility (correctness) and the numerical stability of the instrinsic reconstitution algorithm.

If time permits and pending on the audience will, I will briefly explain the mathematical layout of a far reaching non-linear generalization of the intrin- sic reconstitution algorithm that is general enough to account for the case of molecules with symmetries. This generalization is based on novel optimization paradigm, based on ideas from representation theory and invariant theory of finite subgroups in SO (3), called categorical optimization. In a nutshell, in this paradigm, the solution of the computational problem is characterized as an object in a category instead of as an element of a set as in traditional approaches.

This work is part of a collaborative project conducted with Amit Singer (Princeton), Shamgar Gurevich (Wisconsin Madison), Yoel Shkolnisky (Tel-Aviv University) and Fred Sigworth (Yale).

Cynthia Vinzant

Abstract: Convex hulls of curves appear in many fields, including optimization, discrete geometry, statistics, and chemistry. Understanding and computing the facial structure of the convex hull of a curve is generally a difficult problem. It becomes more tractable when the curve comes with a parameterization. We describe a general method for computing the boundaries of convex hulls of these curves and give many examples.

David Conti

Abstract: In the real world of communications, transmitted data tends to be corrupted by some noise within the transmission process. Codes are concrete mathematical objects that find application as means to encode data in such a way that a bounded amount of errors caused by transmission can be corrected by a decoding algorithm. How to achieve this in the best way is the subject of coding theory, which interestingly leads to a variety of deep mathematical structures. The central paradigm in modern coding theory is to represent codes by special graphs from which powerful decoding algorithms can be derived. Trellises are among the most notable of such graph representations of codes, one important reason being that they benefit of a fruitful algebraic/combinatorial theory. Their decoding performance under modern algorithms has been shown to be linked to so called pseudocodewords. In this talk we will take a stroll through trellises and their pseudocodewords, by presenting some important problems and a motivating conjecture on a special trellis representing the famous Golay code. We will show how an algebraic framework can be developed to help us studying trellis pseudocodewords, bringing into the picture recurrence sequences, symmetric functions, and invariant theory.

Vitaly Skachek

Abstract: Error-correcting codes are used for correction of data errors in communications and information storage devices. Linear error- correcting codes are defined as null-spaces of the corresponding parity-check matrices. Low-density parity-check (LDPC) code is a linear code with a sparse parity-check matrix.

Recently, LDPC codes with their respective message-passing decoding algorithms became a standard for correction of errors in a wide variety of systems. It was observed that adding additional (redundant) rows to a parity-check matrix can improve the performance of the LDPC code. In order to explain this phenomenon, the graph-cover pseudocodewords and their corresponding pseudoweights were introduced by R. Koetter and P. Vontobel.

In this work, we define the BEC, AWGN, BSC, and max-fractional pseudocodeword redundancy $\rho(C)$ of a code C as the smallest number of rows in a parity-check matrix such that the minimum pseudoweight (over the corresponding channel) is equal to the minimum Hamming distance of C. It was shown by Schwartz and Vardy that for any binary linear code, the BEC pseudocodeword redundancy is finite. In this work we show that most binary linear codes do not have a finite AWGN, BSC and max-fractional pseudocodeword redundancy. We also provide bounds on the pseudocodeword redundancy for some families of codes, including codes based on designs. Finally, we obtain the exact values of AWGN and BSC pseudocodeword redundancies for some short codes.

Joint work with Jens Zumbragel and Mark F. Flanagan.

Pablo Parrilo

Abstract:

Risi Kondor

Abstract: The quadratic assignment problem (QAP) is a central problem in combinatorial optimization. Several famous computationally hard tasks, such as graph matching, partitioning, and the traveling salesman all reduce to special cases of the QAP.

We propose a new approach to the QAP based on non-commutative Fourier analysis on the symmetric group. Specifically, we present a branch-and-bound algorithm that performs both the branching and the bounding steps in Fourier space.

By exploiting the band-limited nature of the QAP objective function and using FFT techniques, the algorithm runs in $O(n^3)$ time per branch-and-bound node. The techniques underlying the algorithm generalize to a range of other combinatorial optimization problems.

Deepti Pachauri

Abstract: See http://www.math.wisc.edu/ boston/LinearAlgebraAbstract.pdf

Ben Recht

Abstract: Stochastic Gradient Descent (SGD) is a popular optimization algorithm for solving data-driven problems including support vector machines, text labeling, and matrix factorization. SGD is well suited to processing large amounts of data due to its robustness against noise, rapid convergence rates, and predictable memory footprint. Nevertheless, scaling SGD to very large problems is challenging as SGD seems to be impeded by many of the classical barriers to scalability: (1) SGD appears to be inherently sequential, (2) SGD assumes uniform sampling from the underlying data set resulting in poor locality, and (3) current approaches to parallelize SGD require performance-destroying, fine-grained communication.

This talk aims to refute the conventional wisdom that SGD inherently suffers from these scalability impediments. Specifically, using novel theoretical analysis, algorithms, and implementation, I will show that SGD can be implemented in parallel with minimal communication and without any locking or synchronization. I will also discuss how to induce additional data-locality through biased random orderings of the incremental gradients.

This work raises a series of challenging questions in non-commutative algebraic geometry. Sharpening our convergence analysis depends on controlling the norm of long, random products of matrices. I will discuss how our adaptive ordering schemes give rise to difficult word problems relating arithmetic and geometric means of matrices.

Joint work with Feng Niu, Christopher Re, and Stephen Wright.

Oded Schwartz

Abstract: Algorithms have two kinds of costs: arithmetic and communication. By communication we mean either moving data between levels of a memory hierarchy (in the sequential case) or over a network connecting processors (in the parallel case). The communication of an algorithm often costs significantly more time (and energy) than its arithmetic. Lower bounds on the communication costs thus provide performance scalability limits and motivate the search for communication minimizing algorithms.

Several communication minimizing algorithms and lower bounds were discovered, starting in the late 60's. Recently, lower bounds for many dense linear algebra algorithms have been shown. These were followed by the discovery of corresponding communication minimizing algorithms.

In one approach, we show that the communication costs of algorithms is closely related to the expansion properties of the corresponding computation graphs. We demonstrate this on Strassen's fast matrix multiplication and

other algorithms, and obtain the first lower bounds on their communication costs, for both sequential and parallel models. These bounds are optimal for the sequential case, as they are attainable by existing natural implementations. We use the graph-expansion approach to devise implementations that attain the communication costs lower bounds for the parallel model as well.

Based on joint work with Grey Ballard, James Demmel, Olga Holtz, and Eran Rom.

Lek-Heng Lim

Abstract: Suppose a large number of voters have each rated or compared a small subset of a large number of alternatives, how could we rank the alternatives based on these data? The rank aggregation problem is fraught with famous difficulties – Arrow's impossibility, Saari's chaos, NP-hardness of Kemeny optima. To complicate matters further, let's say the ratings do not come all at once but trickles in on a daily basis and we would like to regularly update our ranking. Let say we also want a measure of reliability or quality of our ranking. We will disucss a method based on Hodge decomposition that meets all these requirements.

Olga Holtz

Abstract: