

**MATH 845: HOMEWORK 4, DUE APR 7.**

1. (a) Let  $K = \mathbf{Q}(\sqrt{m})$  where  $m$  is squarefree and  $|m| \geq 3$ . Show that the narrow class number  $h_+$  of  $K$  is even except possibly when  $m \equiv 1 \pmod{4}$  and  $|m|$  is prime. Let  $L$  be the narrow class field of  $K$ . Show that if  $m$  has at least two distinct odd prime divisors  $p, q$ , then the Galois group of  $L/K$  maps onto the Klein four-group except possibly when  $m \equiv 1 \pmod{4}$  and  $|m| = pq$ .

(b) Show that if  $m = 223$ , then  $h_+ = 6$ .

2. (a) Let  $L/K$  be an extension of number fields and suppose that some prime of  $K$  is totally ramified in  $L/K$ . Show that the class group of  $L$  maps onto the class group of  $K$ , and that the narrow class group of  $L$  maps onto the narrow class group of  $K$ .

(b) Let  $K$  and  $L$  be the  $m$ th and  $n$ th cyclotomic fields respectively, where  $m$  is a divisor of  $n$ . Show that the class group of  $L$  maps onto the class group of  $K$ .