9. If $G$ is a finite group, show that there is a nilpotent subgroup $H$ such that $H^G = G$.

10. Show that if $F$ is any field, then its additive group $F^+$ is characteristically simple, but that $\mathbb{Z}^+$ is not characteristically simple.

11. Show that if $G$ is characteristically simple, then so is $G \times G$.

12. Suppose $H$ and $K$ are subnormal subgroups of $G$. Show that $H \cap K$ is subnormal in $G$. Show by example that $HK$ need not be a subgroup of $G$ but that if $H$ is normal in $G$, then $HK$ is subnormal in $G$. 