17. Prove that a finite $p$-group has maximal class if and only if it has an element whose centralizer has order $p^2$.
HINT: Show that the property of having such an element is inherited by quotient groups of order $\geq p^2$.

18. If $G$ is a finite $p$-group, define $\Omega_i(G) := \langle x \in G : x^{p^i} = 1 \rangle$. Consider the so-called $\Omega$-series of $G$, namely

$$1 = \Omega_0(G) \leq \Omega_1(G) \leq \cdots \leq \Omega_e(G) = G$$

where $p^e$ is the exponent of $G$. Are these always proper inclusions? For "regular" $p$-groups, $\Omega_i(G) = \{ x \in G : x^{p^i} = 1 \}$. Give an example of a finite $p$-group that is not regular.

19. If subgroup $H$ of $G$ satisfies $H' = G'$, show that $L_i(H) = L_i(G)$ for all $i \geq 2$.
HINT: You can use without proof the Three-Subgroup Lemma, which says that if $H, K, L, N$ are subgroups of $G$, with $N$ normal, then if $[H, K, L]$ and $[K, L, H] \leq N$, then $[L, H, K] \leq N$.

20. If $G$ is a $p$-group of order $p^n$ and $\alpha \in \text{Aut}(G)$ has order $p^m$, show that $m < n$. 