Math 114 (Algebra & Trigonometry) 
Lec. 2, Fall Sem. 2001-02 
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NAME: 

TA: 

Solutions of Exam 3

1. If \( u \) is between \( 3\pi/2 \) and \( 2\pi \) and \( \cos u = 0.3 \), determine

   (i) \( \sin u \) exactly: We have \( \sin^2 u = 1 - \cos^2 u \). Since \( u \) is in the 4th quadrant, \( \sin u \) is negative. So
   \[
   \sin u = -\sqrt{1 - \cos^2 u} = -\sqrt{1 - (0.3)^2} = -\sqrt{0.91}.
   \]

   (ii) \( \sin(u/2) \) exactly: The angle \( u/2 \) is in the 2nd quadrant. So
   \[
   \sin(u/2) = +\sqrt{(1 - \cos u)/2} = +\sqrt{(1 - 0.3)/2} = +\sqrt{0.35}.
   \]

   (iii) \( \csc(\pi/2 + u) \) exactly: We have
   \[
   \csc(\pi/2 + u) = \frac{1}{\sin(\pi/2 + u)} = \frac{1}{\cos u} = \frac{1}{0.3}.
   \]

2. Find all solution of:

   \[
   \sin x + 1 = \cos x.
   \]

   Squaring both sides and using identity 1., we get,
   \[
   \sin^2 x + 2 \sin x + 1 = \cos^2 x = 1 - \sin^2 x.
   \]

   Simplifying this gives \( \sin^2 x + \sin x = \sin x(\sin x + 1) = 0 \). Thus either \( \sin x = 0 \), that is,
   \( x = 0 + 2k\pi \) or \( \pi + 2k\pi \) (k any integer), or \( \sin x + 1 = 0 \), equivalently, \( \sin x = -1 \), that is
   \( x = 3\pi/2 + 2k\pi \). Checking we find that 0 and \( 3\pi/2 \) are solutions of the original equation, but \( \pi \) is not. So the answer is
   \[
   x = 0 + 2k\pi \text{ or } 3\pi/2 + 2k\pi, (k \text{ any integer}).
   \]

3. Two trains from Chicago are bound for Madison and St. Louis, respectively. At a certain time, the Madison train has traveled 75 miles, and the St. Louis train has traveled 125 miles. If Chicago to Madison is 30 degrees \textbf{counterclockwise from north}, and Chicago to St. Louis is 45 degrees \textbf{clockwise from south}, how far apart are the two trains at this time?

   Drawing a picture (always do this!) and using the law of cosines, we get that the answer is
   \[
   a^2 = 75^2 + 125^2 - 2(75)(125)\cos 105^\circ
   \]

   which is about 26102, and so \( a \) is about 161.56.
4. (i) (a) The **domain** of the inverse sine function is: \([-1, 1]\), and the **range** is: \([-\pi/2, \pi/2]\).
(b) The **domain** of the inverse tangent function is: \((-\infty, \infty)\), and the **range** is: \((-\pi/2, \pi/2)\).
(c) Sketch the graph of the inverse sine function:

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(ii) Write the complex number \(-4 - 4i\) in **trigonometric form** with angle argument between 0 and \(2\pi\) and then compute its **square roots**, that is, find all solutions of \(u^2 = -4 - 4i\) in **simplest form**.

The complex number \(z = -4 - 4i\) is in the 3rd quadrant. Its modulus is \(\sqrt{(-4)^2 + (-4)^2} = \sqrt{32} = 4\sqrt{2}\). Its argument is a third quadrant angle whose reference angle has tangent equal to \(-4/(-4) = 1\). Thus the reference angle is \(\pi/4\) and the argument of \(z\) is \(5\pi/4\):

\[
z = 4\sqrt{2}(\cos\frac{5\pi}{4} + i \sin \frac{5\pi}{4}).
\]

The square roots are

\[
\sqrt{4\sqrt{2}(\cos\frac{5\pi}{8} + i \sin \frac{5\pi}{8})}, \text{ and } \sqrt{4\sqrt{2}(\cos\frac{13\pi}{8} + i \sin \frac{13\pi}{8})}.
\]

5. Verify to be an identity:

\[
\frac{\sin 2u}{1 + \cos 2u} = \frac{\sin u}{\cos u}.
\]

Using the double angle formula obtained from the addition formula (identity 3.) and identity 1., we get

\[
\frac{\sin 2u}{1 + \cos 2u} = \frac{2\sin u \cos u}{1 + \cos^2 u - \sin^2 u} = \frac{2\sin u \cos u}{2\cos^2 u} = \frac{\sin u}{\cos u}.
\]