1. (10 points) There are 50 A’s, 50 B’s, 50 C’s, and 50 D’s in a hat. If I pick a letter at random out of the hat every second, how long before I am guaranteed of having 15 identical letters?

\[4 \times 14 + 1 = 57\]

2. (10 points) Construct a permutation of \{1, 2, 3, 4, 5, 6, 7, 8, 9\} with inversion sequence 3, 4, 4, 0, 2, 2, 1, 1, 0.

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3. (20 points) Which binary 9-tuple immediately precedes and immediately follows the 10-tuple 0, 1, 1, 0, 0, 1, 1, 1, 0 in each of the base 2 generating scheme order of all binary 10-tuples, reflected Gray code order for all binary 10-tuples, and lexicographic order of binary 10-tuples with 5 1’s?

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Immediately Precedes</th>
<th>Immediately Follows</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base 2 Gen. Scheme</td>
<td>0 1 1 0 0 1 1 1 0 0</td>
<td>0 1 1 0 0 1 1 1 1 0</td>
</tr>
<tr>
<td>Reflected Gray Code</td>
<td>0 1 1 0 0 1 1 1 1</td>
<td>0 1 1 0 0 1 1 1 0 0</td>
</tr>
<tr>
<td>Lex. Order with 6 1’s</td>
<td>0 1 1 0 0 1 1 1 0</td>
<td>0 1 1 0 0 1 1 0 1 1</td>
</tr>
</tbody>
</table>

4. (15 points) (a) What does \(r(3, 4) = 9\) mean?

If the segments of \(K_9\) are colored red or green then one is guaranteed that either there is a red \(K_3\) or a blue \(K_4\). It is possible to color the segments of \(K_8\) red or green creating neither a red \(K_3\) nor a blue \(K_4\).

(b) How many ways are there to separate 25 identical coins into 5 piles so that each pile contains at least 2 coins?

Number of solutions in nonnegative integers of \(y_1 + y_2 + y_3 = y_4 + y_5 = 15\) and so \(\binom{19}{4}\)

5. (30 points) Eighteen (18) different students are to be partitioned into 4 teams: the red team, the blue team, the green team, and the orange team.

(a) How many different partitionss are possible if the size of each team can be anything from 0 to 18?
(b) How many different partitions are possible if the red team contains 5 students, the blue team contains 3 students, the green team contains 6 students, and the orange team contains 4 students?

$$18!/(5!3!6!4!)$$

(c) Now suppose that the 18 students are to be lined in in a red row, a blue row, a green row, and an orange row. How many ways are there to do this?

(This is like problem Page 81, # 45 (c))

$$21!/3!$$

6. (15 points) Give a combinatorial proof of

$$\sum_{k=2}^{n} k(k-1) \binom{n}{k} = n(n-1)2^{n-2}.$$ 

RHS counts the number of committees (of size at least 2) from $n$ people in which each committee has a pres. ($n$ choices), a vice-pres. different from the pres. ($n-1$ choices) filling out the committee ($2^{n-2}$ choices).

LHS counts the same thing by first choosing a committee of size $k$ from 2 to $n$, and then designating one member as pres. ($k$ choices) and a different one as vice-pres. ($k-1$ choices