1. Functions

   (i) Let $A = \{a, b, c, d, e\}$. Draw the digraph of the function $f : A \rightarrow A$ which has the constant value $c$ on $A$.

   (ii) Let $A = \{a, b, c, d, e\}$. Draw the digraph of the identity function on $A$.

   (iii) Let functions $f, g, h, k$ with domain $\mathbb{Z}^+$ be defined by:

   $f(n) = n^3 - 4n^2 + 3$, $g(n) = n^3 + 5n^2$, $h(n) = \log_2(n)$, and $k(n) = \left\lfloor \frac{n + 1}{5} \right\rfloor$.

   Draw the digraph of the relation big-$O$ on $\{f, g, h, k\}$. 
(iv) Consider the permutations
\[
p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 6 & 1 & 2 & 3 & 4 \end{pmatrix}
\quad \text{and} \quad q = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 4 & 3 & 5 & 2 & 1 \end{pmatrix}.
\]

(iv-a) Compute \( p \circ q \)

(iv-b) Compute \( q^{-1} \)

(iv-c) Write \( p \) as a product of disjoint cycles, and as a product of transpositions.

\begin{itemize}
  \item cycles:
  \item transpositions
\end{itemize}

2. Partial Orders

(i) Circle the properties that every finite poset has:
not reflexive – reflexive – symmetric – not symmetric –
antisymmetric – transitive – not transitive –
every pair of elements comparable –
LUB of every pair of elements exists –
maximal elements always exist – a greatest element always exists.
(ii) Consider a six letter alphabet with letters given in order from first to last by $u, v, w, x, y, z$. Put the following “words” in lexicographic order:

wuxy, wuxx, uxzyz, yxx, zu.

(iii) Draw (carefully!) the Hasse diagram of the poset of all subsets of $A = \{a, b, c\}$ partially ordered by set-inclusion. Be sure to label all the points in the diagram.

(iv) List ALL the topological sortings of the poset whose Hasse diagram is:
(v) In the poset of integers \( \{1, 2, 3, \ldots, 15\} \) ordered by divisibility (is a factor of), compute the following or say that it does not exist:

- GLB \( \{10, 14\} \):
- GLB \( \{7, 9\} \):
- LUB \( \{3, 7\} \):
- LUB \( \{3, 4\} \):

(vi) Consider the poset of partitions of \( A = \{1, 2, 3, 4, 5, 6, 7, 8\} \) (i.e. equivalence relations \( R \) on \( A \) partially ordered by set-inclusion), and the partition \( \pi : \{1, 2, 3\}, \{4, 5, 6\}, \{7, 8\} \) with 3 parts. Determine:

(vi-a) a partition less than \( \pi \) with 5 blocks.

(vi-b) a partition greater than \( \pi \) with 2 blocks.

3. **Boolean algebras and Boolean functions**

(i) Is the lattice \( D_{70} \) of all positive divisors of 70 a **Boolean algebra**? Explain/Justify your answer.
(ii) Consider the Boolean function $f$ of 3 Boolean variables $x, y, z$ with $f$ having the value 1 only for $x = 1, y = 0, z = 1$, $x = 0, y = 0, z = 1$, and $x = 1, y = 1, z = 1$. Give the function $f$ as a Boolean expression (polynomial).

4. Trees and Graphs

(i) Construct the rooted tree corresponding to the algebraic expression:

$(((a \times b) - c) \times d) \times (e + (f \times g))$.

(ii) List the elements of the rooted tree in (i) in preorder:

(iii) Evaluate the following algebraic expression given in Polish (prefix) form:

$x \times x + (3) (4) (5) (6) + x (7) (8) (9)$
(iv) Represent the following ordered tree as a binary positional tree:

(v) Consider the weighted graph below to which we have begun to apply either Prim’s algorithm or Kruskal’s algorithm (those edges bolded are the ones picked thus far).

(v-a) Which algorithm was applied in this case (or could it have been either)?

(v-b) What is the next edge that can be chosen using the algorithm?

(vi) what is the chromatic number and chromatic polynomial of the (undirected) graph which is a cycle with 4 vertices and 4 edges?