1. (20 points) We have a multiset $X$ of 14 balls, consisting of 3 identical Red, 4 identical Blue, 5 identical Green, and 2 identical Yellow balls.

(a) How many (linear) permutations of $X$ are there?

(b) How many (linear) permutations of $X$ are there if the yellow balls are not consecutive?

(c) How many circular permutations of $X$ are there if the Yellow Balls are opposite one another?

(d) How many ways are there to arrange the balls of $X$ in a 2 by 7 formation if the Yellow Balls are to be in different rows?
2. (10+5=15 points)
   (a) Evaluate the following sum: \( \sum_{k=1}^{n} (-1)^{k-1} \binom{n}{k} \) for \( n \geq 1 \).

   (b) Explain the combinatorial reasoning behind the recurrence relation for the Stirling numbers of the second kind:

   \[ S(p, k) = kS(p - 1, k) + S(p - 1, k - 1) \quad (1 \leq k \leq p - 1). \]
3. (15 points) How many ways are there to place 7 non-attacking rooks on the 7 by 7 board with forbidden positions as shown:

\[
\begin{array}{ccccccc}
X & X & & & & & \\
X & X & & & & & \\
X & X & X & & & & \\
X & X & X & & & & \\
X & X & X & & & & \\
X & X & & & & & \\
X & X & & & & & \\
\end{array}
\]

4. (15 points) Identify each of the relations on a set X below as a partial order, equivalence relation, total order, or none of the above:

(a) X the collection of all subsets of \(\{1, 2, \ldots, 10\}\) with \(A R B \) iff \(A \subseteq B\).

(b) X the set of all real numbers with \(A R B \) iff \(|a| = |b|\).

c) X the set of positive integers with \(a R b \) iff \(a\) is a factor of \(b\).

(d) X the set of vertices of a tree \(T\) with root \(r\) with \(a R b \) iff the chain from \(r\) to \(b\) in \(T\) passes through \(a\).

(e) X the set of ordered pairs \((a, b)\) of real numbers with \((a, b) R (c, d) \) iff \(a \leq c\) (as real numbers) and \(b \leq d\).
5. (10 points) Give the ordinary generating function in closed form for the number $h_n$ of solutions in nonnegative integers of the equation $3e_1 + e_2 + 2e_3 + 5e_4 = n \ (n \geq 0)$.

6. (15 points) Determine the chromatic polynomial of the graph:
7. (10 points) Consider the following network $N$ with source $s$ and target $t$ where the numbers on arcs represent their capacities and the numbers in brackets $[·]$ on arcs represent the value of a function $f$ on the arcs of $N$.

(a) Check that $f$ is a flow from $s$ to $t$, and determine its value:

(b) Starting with $f$, use the basic flow algorithm and obtain a "flow-augmenting path" from $s$ to $t$ to give a flow $f'$ whose value is one more than the value of $f$. Is $f'$ a maximum flow? If so, give a cut whose capacity equals the value of $f'$. 
8. (15 points) Use **Burnside’s Theorem** to determine the number of inequivalent colorings of the corners of a regular hexagon (6-gon) $P$ in the presence of the full corner symmetry group of $P$. 
9. (10 points) A *marked 4-omino* is a 1 by 4 piece of 4 unit squares joined side to side where each square is marked with 1, 2, 3, 4, 5, or 6 dots. Use **Burnside’s Theorem** to determine the number of different marked 4-ominoes.
EXTRA CREDIT PROBLEM (20 points) Consider the graph $G$ with vertices and edges as shown:

(a) How many chains of length 12 connect the lower left vertex $X$ with the upper right vertex $Y$?

(b) How many such chains are there if the "middle vertex" $A$ and all the edges that touch it are eliminated from the graph?

(c) How many such chains which do not use any of the vertices on the diagonal running from $X$ to $Y$?