

Preface

In the Preface for the third edition of *Introductory Combinatorics* I related how I extensively rewrote some sections and included some new material and exercises. Some of the major changes from the second to third editions were given as:

An introductory section on partial orders and equivalence relations has been added to Chapter 4.

Chapter 5 contains a new section on partially ordered sets, where Dilworth's theorem and its dual are proved.

A section on partitions of a positive integer has been added to Chapter 8.

In Chapter 11, the first chapter on graph theory, a tree is now defined as a connected graph that becomes disconnected upon the removal of any edge, and the section on digraphs has been removed.

Chapter 12, which is new, discusses digraphs and networks. This chapter includes a proof of the max-flow min-cut theorem of Ford and Fulkerson, from which Menger's theorem and König's theorem of Chapter 9 are deduced as corollaries.

Fundamental numbers of graph theory, discussed in Chapter 12 in the second edition, are now Chapter 13.

Pólya counting, formerly Chapter 13, is now Chapter 14.

For this fourth edition, I corrected all of the typos I knew of, made some minor adjustments in language (including, in the chapters on graph theory, replacing "chain" with "path"), inserted some small additions, and added more than 60 new, challenging exercises. I hesitated to change the book too much or add too many new topics. I don't like books that have "too many words" (and this preface will not have too many words), and I don't want to fall into that trap. Nevertheless, I added two new sections. In Chapter 6, I have added a new last section on Möbius inversion as a generalization of the inclusion-exclusion principle. In Chapter 8, I have added a new section on lattice paths and the small and large Schröder numbers.

As with earlier editions, one can use this book for either a one or two semester undergraduate course. A first semester could have an

emphasis on counting and a second semester an emphasis on graph theory and designs. One could also put together a one semester course which does some counting and graph theory, or some counting and design theory. A brief commentary on each of the chapters and their interrelation follows:

Chapter 1 is an introductory chapter; I usually select one or two topics from it and spend at most two classes on this chapter. Chapter 2, on the pigeonhole principle, should be discussed at least in abbreviated form. But note that no use is made later of some of the harder applications of the pigeonhole principle and of the section on Ramsey's theorem. Chapters 3 to 8 are primarily concerned with counting techniques and properties of some of the resulting counting sequences. They should be covered in sequence. Chapter 4 is about schemes for generating permutations and combinations and, as mentioned above, includes an introduction to partial orders and equivalence relations. However, except for the section on partially ordered sets in Chapter 5, chapters beyond Chapter 4 are essentially independent of Chapter 4, and so this chapter can either be omitted or abbreviated. And one can decide not to cover partially ordered sets at all. Chapter 5 is on properties of the binomial coefficients, and Chapter 6 covers the inclusion-exclusion principle. The new section on Möbius inversion is not used in later sections. Chapter 7 is a long chapter on solving recurrence relations and the use of generating functions in counting. Chapter 8 is concerned mainly with the Catalan numbers, the Stirling numbers of the first and second kind, partition numbers and, now in this latest edition, the large and small Schröder numbers. The chapters that follow are independent of it.

Chapter 9 concerns matchings in bipartite graphs. I introduce bipartite graphs before graphs, but there is no essential dependence of this chapter on the later chapters on graph theory. Except for the application of matching theory to Latin squares, Chapter 10 on designs is independent of the rest of the book. Toward the end of section 10.4, I make use of the matching theory developed in Chapter 9. Chapters 11 and 13 contain an extensive discussion of graphs, with some emphasis on graph algorithms. Chapter 12 is concerned with digraphs and network flows. Chapter 14 deals with counting in the presence of the action of a permutation group and does make use of many of the earlier counting ideas. Except for the last example, it is independent of the chapters on graph theory and designs.

When I teach a one semester course out of this book, I like to conclude with Chapter 14 on Pólya counting as this theory enables one to solve many counting problems that can't be touched with the techniques of earlier chapters. Following Chapter 14, I give solutions and hints for some of the approximately 650 exercises in the book. A few of the exercises have a * symbol beside them, indicating that they are more challenging. The end of a proof and the end of an example are indicated by writing a \square symbol.

It is difficult to assess the prerequisites for this book. As with all books intended as textbooks, having highly motivated and interested students helps. Perhaps the prerequisites can be best described as the mathematical maturity achieved by the successful completion of the calculus sequence and an elementary course on linear algebra. Use of calculus is minimal, and the references to linear algebra are few and should not cause any problem to those not familiar with it.

It is very gratifying for me that after more than 25 years since the first edition of *Introductory Combinatorics* was published, it continues to be well-received by the professional mathematical community.

I am very grateful to many individuals who encouraged me to do a fourth edition and who provided me with useful comments: Russ Rowlett (UNC, Chapel Hill), James Sellers (Penn State University), and Michael Buchner (Univ. of New Mexico). As in the third edition I want to especially acknowledge Leroy F. Meyers and Tom Zaslavsky, each of whom provided me with extensive and detailed comments (for the third edition). For the fourth edition, I received many useful comments from Nils Andersen (Univ. of Copenhagen), James Propp (Univ. of Wisconsin), and Louis Deaett (Univ. of Wisconsin), who read and commented on the new section on lattice paths. The book, I hope, continues to reflect my love of the subject of combinatorics, my enthusiasm for teaching it, and the way I teach it.

Finally, I want to thank again my dear wife, Mona, who continues to bring such happiness, spirit, and adventure into my life.

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