Math 221 Section 301 & 302
Midterm II Review

Review sheet of some of the topics covered in the second midterm with practice problems.

1 Exam Tips

• Make sure you carefully read the statement of each question and answer what is asked of you.
• Before going into some long calculation, always see if you can use some algebra to simplify the problem first.
• If you’re going to use a theorem, first make sure that all the conditions of the theorem are satisfied.
• Don’t spend too much time on any one problem. If you get stuck on a problem, move on and try to come back to it later.
• Be neat. Sloppy work is indicative of a sloppy thought process. Messy work is more error prone and makes graders angry.

2 Questions you should know the answer to:

1. What do each of the following theorems say?
   • Intermediate Value Theorem
   • Mean Value Theorem
   • Extreme Value Theorem

2. What is implicit differentiation and when do you use it?

3. How do you sketch the graph of a function? In particular:
   • What are critical points and how can you determine when a critical point is a local maximum or minimum? How do you find the global maximum or minimum of a function?
   • How can you determine where a function is increasing or decreasing?
   • What is an inflection point? How do you determine where a function is concave or convex?
   • How do you determine if a function has any vertical, horizontal, or slanted asymptotes?

4. How do you go about solving applied optimization problems?

5. What does l’Hopital’s rule say and when can you apply it?

6. What is a parametrized curve? How do you find the line tangent to a parametrized curve at a given point? How do you find the curvature?

7. What are the basic limits involving exponential or logarithmic functions?

8. How do you solve exponential growth or decay problems?

9. What is a Riemann sum? What is a definite integral?

10. What is an antiderivative and how do you compute antiderivatives of simple functions?

11. What does the Fundamental Theorem of Calculus say?
3 Practice Problems

1. Calculate the derivatives of the following functions:
   (a) \( f(x) = \ln\left( \frac{1 + x^2}{1 - x^2} \right) \),
   (b) \( f(x) = \sqrt{1 + e^{-x}} \),
   (c) \( f(x) = \ln(x^e) \).

2. Find the formula for the \( k \)th derivative of \( f(x) = xe^x \).

3. Find the equation of the tangent line to the “astroid” curve defined implicitly by
   \[ x^{2/3} + y^{2/3} = 4 \]
   at the point \((-\sqrt{27}, 1)\).

4. A particle is moving along a vertical axis so that its position \( y \) (in meters) at time \( t \) is given by the equation
   \[ y(t) = t^3 - 3t + 3, \quad t \geq 0. \]
   Determine the total distance traveled by the particle in the first three seconds.

5. What are the dimensions of the largest rectangle that can be inscribed in the ellipse \( x^2 + 4y^2 = 16 \)?

6. Calculate the smallest possible product of two numbers, if one is exactly 9 greater than the other.

7. Find the following limits.
   (a) \( \lim_{t \to 0} \frac{\sin(t^2)}{t^2} \),
   (b) \( \lim_{x \to -2} \frac{x^3 + 8}{x + 2} \),
   (c) \( \lim_{x \to \infty} \left( (x^3 + 5x^2 - 3x - 9)(e^{-3x}) \right) \),
   (d) \( \lim_{x \to \pi/2} (\sec(x) - \tan(x)) \),
   (e) \( \lim_{x \to \infty} \frac{e^x - x^2}{e^x + x} \).

8. Sketch the graphs of the following functions (and do all the stuff about finding signs of \( f \), \( f' \), \( f'' \), stationary points, asymptotes, inflection points, maxima/minima, etc):
   (a) \( y = x^2e^{x^2} \),
   (b) \( y = e^{x/2} \),
   (c) \( y = \ln(1 + x^2) \),
   (d) \( y = \sqrt{1 - x^2} \).

9. Consider the parametrized curve given by \( x(t) = 3 \cos t, y(t) = 4 \sin t \).
    • Find the equation for the line tangent to the curve when \( t = \pi/2 \).
    • Calculate the curvature where \( t = \pi/2 \).

10. Carbon-14 has a half life of 5,730 years. Given a sample with an initial mass of 300g, what is the remaining mass after 250 years have elapsed?

11. Calculate the area bounded by the graphs of \( y = x + 3 \) and \( y = x^2 - x \).

12. Calculate the average value of \( f(x) \) between \( x = 0 \) and \( x = 4 \).