1. Find the largest possible domain of the function

\[ f(x) = \frac{1}{\sin x}. \]

(Not for points bonus: What is the range of this function?)

This is well defined for all \( x \) such that \( \sin x \neq 0 \). \( \sin x = 0 \) for \( x = k\pi \) for all integers \( k \). Hence the domain is:

\[ \mathbb{R} \setminus \mathbb{Z}\pi : k \in \mathbb{Z} \}

Since \( \sin x \) takes all values between -1 and 1, the range of \( f(x) \) is \( (-\infty, -1] \cup [1, \infty) \).

2. Find a formula for the function \( g \) that is defined by the requirement that

\[ y = g(x) \iff y^2 - x^2 = 0 \text{ and } y \geq 0. \]

Solving \( y^2 - x^2 = 0 \), we get \( y = x \) or \( y = -x \). Since we want \( y \geq 0 \), we choose \( y = x \) if \( x \geq 0 \) and \( y = -x \) if \( x \leq 0 \). Thus the function \( g(x) \) is given by \( g(x) = |x| \).