1. Let $f(x) = \sqrt{x}$. Find the equation for the line tangent to the graph of $f$ at $x = 4$ and determine where the line intersects the $x$-axis.

$$f'(x) = \frac{1}{2\sqrt{x}}.$$ The slope of the tangent line at $x = 4$ is $f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$.

Since $f(4) = 2$, the tangent line passes through the point $(4, 2)$. Thus, the equation of the tangent line is given by $y = \frac{1}{4}(x - 4) + 2$.

This line intersects the $x$-axis when $y = 0$, so we solve $0 = \frac{1}{4}(x - 4) + 2$

to get $x = 8$. Thus, the intersection is at $(8, 0)$.

2. Let $g(x) = x^3 - x$. Determine on which intervals $g$ is positive or negative and on which intervals $g$ is increasing or decreasing.

$$g(x) = x^3 - x = x(x^2 - 1) = x(x + 1)(x - 1).$$ Thus $g$ has zeros at 0, 1, and -1.

$g$ is positive on $(0, 1)$ and $(-1, 0)$, negative on $(-\infty, -1)$ and $(1, \infty)$.

$g'(x) = 3x^2 - 1$. Thus $g'(x) = 0$ when $x = \pm \frac{1}{\sqrt{3}}$.

$g$ is increasing on $(-\infty, -\frac{1}{\sqrt{3}})$ and $(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$, decreasing on $(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$.

3. If you finish early, please draw a math cat.