1. Find the maximum and minimum values of \((1-x)^5 + 5x\) for \(1 \leq x \leq 3\) and give the values of \(x\) where the maximum and minimum occur.

Let \(f(x) = (1-x)^5 + 5x\). Then \(f'(x) = -5(1-x)^4 + 5\).

We can see that \(f'(x) = 0\) when \(x = 2\).

\[f' \quad \begin{array}{c} + \end{array} \quad 1 \quad 2 \quad 3 \quad \text{We can see, 2 is a local max, and the two endpoints are local minima.}\]

Plugging in the values, we see \(f(2) = 9\) is the global maximum and \(f(1) = 5\) is a local min with \(f(3) = -17\) the global min.

2. Suppose that we wish to construct a rectangular box with a square base. If we require that the surface area of the box is 54 square feet, what is the largest possible volume of the box?

Objective function is volume \(V(l,h) = lhl^2\). Constraint is surface area \(54\) square feet. \(2l^2 + 4lh = 54\).

Use constraint to eliminate a variable in the objective function.

\[2l^2 + 4lh = 54 \quad \rightarrow \quad 4lh = 54 - 2l^2 \quad \rightarrow \quad lh = \frac{54 - 2l^2}{4}\]

\[V = lhl^2 = (lh)l = \left(\frac{54 - 2l^2}{4}\right)l\]

Maximize \(V(l) = \frac{54l - 2l^3}{4}\). \(V'(l) = \frac{54 - 3l^2}{2}\). Set \(V'(l) = 0\), \(l = \frac{9}{\sqrt{2}} - \frac{3l^2}{2}\).

Dimensions: \(h = l = 3\). \(V = 27 + 18\) cubic feet. Use constraint to get \(h = 3\).

3. If you finish early, please write down your best piece of trivia or interesting fact.