1. Find the asymptotes of \( f(x) = \frac{\sin x}{x-\pi} \).

We suspect that there might be a vertical asymptote at \( x=\pi \), however, we calculate:

\[
\lim_{x \to \pi} \frac{\sin x}{x-\pi} = \lim_{u \to 0} \frac{\sin(u+\pi)}{u} = \lim_{u \to 0} \frac{-\sin(u)}{u} = -1 \neq 0,
\]

and see that there is not a vertical asymptote there.

To see that \( f(x) \) has a horizontal asymptote of \( y=0 \), we calculate that

\[
\lim_{x \to \infty} \frac{\sin x}{x-\pi} = 0.
\]

2. Use the definition of the derivative to compute the derivative of \( f(x) = x^2 - 2x \).

\[
\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 - 2(x+\Delta x) - (x^2 - 2x)}{\Delta x}
\]

\[
= \lim_{\Delta x \to 0} \frac{x^2 + 2x \Delta x + \Delta x^2 - 2x - 2\Delta x - x^2 + 2x}{\Delta x}
\]

\[
= \lim_{\Delta x \to 0} \frac{2x \Delta x + \Delta x^2 - 2\Delta x}{\Delta x}
\]

\[
= \lim_{\Delta x \to 0} \frac{2x + \Delta x - 2}{\Delta x}
\]

\[
= 2x - 2.
\]

Thus \( f'(x) = 2x - 2 \).
Individual Questions

1. Find the asymptotes of the following functions.

   (a) \( f(x) = \frac{x}{x - 3} \).

   \[ \lim_{x \to 3^+} f(x) = \infty, \quad \lim_{x \to 3^-} f(x) = -\infty \Rightarrow \text{vertical asymptote at } x = 3. \]

   \[ \lim_{x \to \infty} f(x) = 1 \Rightarrow \text{horizontal asymptote at } y = 1. \]

   (b) \( g(x) = \frac{3x^3 - x^2 + 1}{x^2 - 4} \).

   Calculate \( \lim_{x \to 2} g(x) \) and \( \lim_{x \to -2} g(x) \) to show \( \text{vertical asymptotes at } x = 2, x = -2. \)

   We then calculate

   \[ \lim_{x \to \infty} \frac{g(x)}{x} = 3 \text{ and } \lim_{x \to \infty} g(x) - 3x = -1 \text{ to show that} \]

   \[ \text{there is a slanted asymptote of } y = 3x - 1. \]

   (c) \( h(x) = \frac{\sin x (3x + 2)}{x^2 + 3} \),

   This has no vertical asymptotes.

   This has a horizontal asymptote of \( y = 0 \).
2. Using the definition, compute the derivatives of $f(x)$:

(a) $f(x) = \frac{1}{\sqrt{x}}$,

\[
\lim_{\Delta x \to 0} \frac{\frac{1}{\sqrt{x+\Delta x}} - \frac{1}{\sqrt{x}}}{\Delta x} = \lim_{\Delta x \to 0} \frac{\frac{1}{\sqrt{x+\Delta x}} - \frac{1}{\sqrt{x}}}{\Delta x} \cdot \left(\frac{\frac{1}{\sqrt{x+\Delta x}} + \frac{1}{\sqrt{x}}}{\frac{1}{\sqrt{x+\Delta x}} + \frac{1}{\sqrt{x}}}\right) = \lim_{\Delta x \to 0} \frac{x - x - \Delta x}{\Delta x (x+\Delta x)} \cdot \frac{x}{\Delta x (\frac{1}{\sqrt{x+\Delta x}} + \frac{1}{\sqrt{x}})}
\]

\[
= \lim_{\Delta x \to 0} \frac{-1}{x (x+\Delta x) (\frac{1}{\sqrt{x+\Delta x}} + \frac{1}{\sqrt{x}})} = \frac{-1}{x^{3/2}}
\]

(b) $g(x) = \frac{2}{1+x}$,

\[
\lim_{\Delta x \to 0} \frac{\frac{2}{1+x+\Delta x} - \frac{2}{1+x}}{\Delta x} = \lim_{\Delta x \to 0} \frac{2(1+x) - 2(1+x+\Delta x)}{\Delta x (1+x+\Delta x)(1+x)} = \lim_{\Delta x \to 0} \frac{-2\Delta x}{\Delta x (1+x+\Delta x)(1+x)}
\]

\[
= \frac{-2}{(1+x)^2}
\]