Examples:

1. (a) Let $D(t)$ be the distance between the point $P(t)$ and the origin. We then have that

$$D(t)^2 = (x(t))^2 + (y(t))^2.$$ 

Taking derivative w.r.t. $t$, we get that

$$2D(t)D'(t) = 2x(t)x'(t) + 2y(t)y'(t).$$ 

Since $y(t) = x(t)^2$, $y'(t) = 2x(t)x'(t)$, so

$$= 2x(t)x'(t) + 2y(t)(2x(t)x'(t)).$$ 

we are given that $x(t) = 2$, and we are looking at this when $x(t) = 3$, $y(t) = 9$.

Thus $D(t) = \sqrt{3^2 + 9^2}$, so we get

$$2(\sqrt{3^2 + 9^2})D(t) = 2(3)(2) + 2(9)(2(3)(2)).$$

so $D'(t) = \frac{114}{\sqrt{108}}$.

(b) Let $A(t)$ be the area. Then $A(t) = x(t)y(t)$. We wish to find $A'(t)$.

By product rule $A'(t) = x'(t)y(t) + x(t)y'(t) = 2y(t) + 3x(t)y(t)$.

$= 2(9) + 3(2)(3)(2) = 54$.

(c) The slope of the tangent is $y'(t) = 2x(t)x'(t)$. We wish to calculate the rate of change, so we want to find $(y'(t))'$. But $y'(t) = \frac{dy}{dx}(t)$, so $(y'(t))' = \frac{4}{x'(t)} = 4\cdot 2 = 8$. 

(c) \( \angle QOP \) where \( Q = (3,0) \).

Let \( \theta(t) \) be the angle \( \angle QOP \). We then have that

\[
\sin (\theta(t)) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{y(t)}{D(t)}.
\]

We want to find \( \theta'(t) \).

Take derivative \( \Rightarrow \cos (\theta(t)) \theta'(t) = \frac{y(t)x'(t) - x(t)y'(t)}{(D(t))^2} \)

\[
\Rightarrow \theta'(t) = \frac{\frac{q}{\sqrt{3^2 + q^2}}}{\frac{12 \sqrt{2} q^3 - q \left( \frac{11y}{\sqrt{3^2 + q^2}} \right)}{3^2 + q^2}} \cdot \frac{\sqrt{3^2 + q^2}}{q}.
\]

\[
\Rightarrow \theta'(t) = \frac{12 \sqrt{2} q^3 - q \left( \frac{11y}{\sqrt{3^2 + q^2}} \right)}{3^2 + q^2} \cdot \sqrt{3^2 + q^2}.
\]