Math 222
QUIZ 10

Name:
Circle Your Section: 349 350

1. Determine the convergence of the following series. State which test you are using and justify your answer.

(a) \( \sum_{n=1}^{\infty} \frac{n + 4}{n^3 - 5n^2 - 3} \).

Use the limit comparison test: Note that for \( n \geq 6 \), terms are positive.

Let \( a_n = \frac{n + 4}{n^3 - 5n^2 - 3} \) and \( b_n = \frac{n}{n^3} \).

Then \( \lim_{n \to \infty} \frac{a_n}{b_n} = \lim_{n \to \infty} \frac{n + 4}{n^3 - 5n^2 - 3} \cdot \frac{n^3}{n} = 1 > 0 \).

Thus \( \sum_{n=1}^{\infty} a_n \) converges if and only if \( \sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{1}{n} \) converges.

Since \( \sum_{n=1}^{\infty} \frac{1}{n} \) diverges, so does our original series.

(b) \( \sum_{n=0}^{\infty} \frac{1}{n^2 + 1} \).

Integral test. \( f(x) = \frac{1}{1 + x^2} \). This is positive, strictly decreasing from 0 to \( \infty \).

Thus \( \sum_{n=0}^{\infty} \frac{1}{n^2 + 1} \) converges if and only if \( \int_0^{\infty} \frac{1}{1 + x^2} \, dx \) also converges.

\[ \int_0^{\infty} \frac{1}{1 + x^2} \, dx = \lim_{N \to \infty} \arctan(x) \bigg|_0^N = \frac{\pi}{2} \cdot \]

Thus our original series converges.
2. Find the radius and interval of convergence for the power series

\[ \sum_{n=0}^{\infty} \frac{(x-2)^n}{4^n}. \]

Use the ratio test: \[ a_n = \frac{(x-2)^n}{4^n}. \]

\[ \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} \left| \frac{x-2}{4} \right| \left( \frac{1}{|x-2|} \right)^n = \left| \frac{x-2}{4} \right|. \]

If \( \left| \frac{x-2}{4} \right| < 1 \), then \( |x-2| < 4 \). Thus the radius of convergence is 4.

The series converges for \(-2 < x < 6\).

At either endpoint the series diverges.

3. Bonus: Why?

Because.