Math 222
QUIZ 6

Name:
Circle Your Section: 349 350

1. Consider the curve \( y(x) \) to the differential equation \( y' = 1 + xy^2 \) that goes through the point \((1, 1)\).

(a) Is \( y = 1 + x^2 \) a solution to this differential equation?

(b) Use Euler's method with two steps (and a step size of \( h = .1 \)) to approximate \( y \) when \( x = 1.2 \).

(a) Let's check. \( y' = 1 + x^2 \). If \( y = 1 + x^2 \) is a solution, then:

\[
(1+x^2)' = 1 + x (1+x^2)^2 \\
2x = 1 + x (1 + 2x^2 + x^4) \\
2x = 1 + x + 2x^3 + x^5 \\
\text{false}
\]

Thus \( y = 1 + x^2 \) is not a solution.

(b) \[
\begin{array}{c|c|c|c}
 x & y & y' & \Delta y \\
 \hline
 1 & 1 & 2 & (1)(2) = .2 \\
 1.1 & 1.2 & 1 + (1)(.1) & (1)(1 + (1)(1.1)^2) \\
 1.2 & 1.2 + (.1)(1 + (1)(1.1)^2) & \\
 \end{array}
\]

so \( y(1.2) \approx 1.2 + (.1)(1 + (1)(1.1)^2) = 1.2 + .1 + .11(1.2)^2 \)

\[
= 1.4584
\]
2. Rabbits in Madison have a birth rate of 5% per year and a death rate from old age of 2% per year. Each year 1000 rabbits get run over and 700 rabbits move in from Sun Prairie.

(a) Write down the differential equation that describes Madison’s rabbit population at time \( t \).

(b) Find the general solution to this differential equation.

\[(a) \quad P(t) = \text{rabbit population at time } t, \]
\[P' = 0.05P - 0.02P - 1000 + 700\]
\[P' = 0.03P - 300\]

(b) This is a first-order linear DE.

\[P' - 0.03P = -300, \quad a(t) = -0.03, \quad A(t) = \int a(t) \, dt = -0.03t.\]

\[m(t) = e^{A(t)} = e^{-0.03t}\]

\[P(t) = \frac{1}{m(t)} \int m(t) (-300) \, dt\]
\[= e^{0.03t} \int -300 (e^{-0.03t}) \, dt\]
\[= e^{0.03t} \left[ -300 \frac{e^{-0.03t}}{-0.03} + C \right]\]
\[P(t) = 10000 + Ce^{0.03t}\]

3. **Bonus:** I want to read a good book over spring break. What do you recommend?

*You should review your math textbooks over spring break!*