222 Worksheet 14
Topics: Taylor Series, Little-oh Notation

Exercises:
1. Find the Taylor series for \( \arctan x \).
2. If \( f(x) = o(x^4) \), is \( f(x) = o(x^3) \)?
3. Simplify the expression \((1 + x + x^2 + o(x^3))(1 + x^2 + o(x^4))\).
4. Using the Taylor polynomials for \( \sin x \) and \( \cos x \), find the 5th degree Taylor polynomial for \( \sin x \cos x \) (make sure to keep track of the little-oh terms).
5. Determine if the following are true or false:
   (a) \( \sin(x^2) - \sin^2(x) = o(x^3) \).
   (b) \( \cos(x^2) - \cos^2(x) = o(x^3) \).
6. Given that \( \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + o(x^5) \) and \( \tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + o(x^5) \), show that
\[
\lim_{x \to 0} \frac{\sin x - \tan x}{x^3} = -\frac{1}{2}.
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