222 Worksheet 9
Topics: Solving Differential Equations

The two types of differential equations we’ll be studying in this course are separable differential equations and first order linear differential equations.

Examples:

1. How can you tell if a differential equation is separable? How do you solve a separable differential equation?

2. How can you tell if a differential equation is first order linear? How do you solve a first order linear differential equation?

3. Solve the initial value problem \( y' = \frac{xy^3}{\sqrt{1 + x^2}}, \ y(0) = -1 \).

4. Find the general solution to \( xy' = x^2 + 3y, \ x > 0 \).

Exercises: Find the general solution to each of the following separable differential equations. If an initial condition is given, find the corresponding solution.

1. \( \frac{dy}{dx} = 6y^2 x, \ y(1) = \frac{1}{25} \)

2. \( y' = e^{-y}(2x - 4), \ y(5) = 0 \)

3. \( y' = xe^{x^2 - \ln(y^2)} \)

Find the general solution to each of the following first order linear differential equations. If an initial condition is given, find the corresponding solution.

1. \( y' + \frac{y}{x^2} = \frac{2}{x^3} \)

2. \( e^x \frac{dy}{dx} + 2e^x y = 1 \)

3. \( (x + 1)y' - 2(x^2 + x)y = \frac{e^{x^2}}{x + 1}, \ x > -1, \ y(0) = 5 \)

4. \( \theta \frac{dy}{d\theta} = \theta^3 \sec \theta \tan \theta, \ \theta > 0, \ y(\pi/3) = 2 \)

5. \( \tan \theta \frac{dr}{d\theta} + r = \sin^2 \theta, \ 0 < \theta < \pi/2 \)