2. As x increases, the plane tilts downwards, so a < 0. As y increases, the plane slopes upwards, so b > 0.
when x = 0, y = 0, the point z lies above the x, y plane, so z > 0.

3. (a) \( z = 3x - y + 6 \). This intersects the z-axis when \( x, y = 0 \), at \( (0, 0, 6) \).

Intersects x-axis when \( y = 0, z = 0 \), at point \( (-2, 0, 0) \).
Intersects y-axis when \( x = 0, z = 0 \), at point \( (0, 6, 0) \).

(b) \( z = ax + by + c \). Intersects x-axis at \( z = 0 \) \( \Rightarrow c = 0 \).
\( z = ax + by + 3 \) Intersescts x-axis at \( y = 0 \) \( \Rightarrow 0 = a(0) + 3 \Rightarrow a = -\frac{3}{4} \).
\( z = -\frac{3}{4}x + by + 3 \) Intersects y-axis at \( z = 0 \) \( \Rightarrow 0 = b(0) + 3 \Rightarrow b = -\frac{3}{2} \).
\( z = -\frac{3}{4}x - \frac{3}{2}y + 3 \).

4. (45) \( z = ax + by + c \). We can rewrite this as \( -ax - by + z = c \), and see that the point \( P = (0, 0, c) \) is in the plane. Then \( \mathbf{n} = (-a, -b, 1) \) is a normal vector to this plane. A point \( X = (x, y, z) \) is in the plane if and only if \( \mathbf{n} \cdot \mathbf{P} = 0 \).

To determine the distance from the origin to the plane we use the formula:
\[
d = \frac{|\mathbf{n} \cdot \mathbf{P}|}{||\mathbf{n}||} = \frac{|-c|}{\sqrt{a^2 + b^2 + 1}} = \frac{c}{\sqrt{a^2 + b^2 + 1}}.
\]
5. \( f(x,y) = x^2 + 2y^2 \). \( x^2 \) and \( 2y^2 \) \( \geq 0 \) for any \( x, y \). The zero set is only the origin \( x=0, y=0 \), and \( f \) is otherwise positive, so \( f(x,y) \) is positive definite.

(b) \( Q(x,y) = x^2 - y^2 = (x-y)(x+y) \). This has the zero set \( x=y \) and \( x=-y \). \( Q \) is positive when \( (x-y)>0, (x+y)>0 \) or when \( (x-y)<0 \) and \( (x+y)<0 \).

(c) \( g(x,y) = x^2 - 4xy + 3y^2 = (x^2 - 4xy + 4y^2) - y^2 = (x-2y)^2 - y^2 = (x-2y-y)(x-2y+y) = (x-3y)(x-y) \).

\( g \) has zero set \( x=3y \) and \( x=y \). Positive when \( (x-3y)>0, (x-y)>0 \) or when \( (x-3y)<0, x-y <0 \). Negative when \( (x-3y)>0, (x-y)<0 \) or when \( (x-3y)<0, (x-y)>0 \).

(d) \( Q(s,t) = 9s^2 - 36st + 36t^2 = 9(s^2 - 4st + 4t^2) = 9((s-2t)^2 + 4t^2) \). \((s-2t)^2 \geq 0\) and \( 4t^2 \geq 0 \) for all \( s, t \). \( Q \) is zero \( \iff (s-2t)^2 = 0 \) and \( 4t^2 \geq 0 \) \( \iff s = 0, t = 0 \). So the origin is the only zero of \( Q \), otherwise \( Q \) is positive, so \( Q(s,t) \) is positive definite.
(e) \( M(\alpha, \beta) = \frac{1}{2} \alpha^2 - \alpha \beta + \beta^2 = \frac{1}{2} (\alpha^2 - 2 \alpha \beta + 2 \beta^2) = \frac{1}{2} (\alpha^2 - 2 \alpha \beta + \beta^2 + \beta^2) \)
\[ \quad = \frac{1}{2} (\alpha \beta + \beta^2). \]

Same as (d), two at origin, \([\text{positive definite!}]\)

(f) \( Q(x, y) = xy + y^2 = y(y + \alpha). \)
This is zero when \( y = 0 \) or when

\[ y + x = 0, \quad y = 0, \quad y(x + \alpha) = 0. \]

Positive when \( y > 0, y + x > 0 \)

or \( y < 0, y + x < 0. \)

Negative when \( y > 0, y + x < 0 \)

or \( y < 0, y + x > 0. \)

\([\text{Indefinite!}]\)

(g) \( Q(x, y) = x^2 + 2xy = x(x + 2y). \) Zero when \( x = 0 \) or \( x + 2y = 0. \)

Positive when \( x > 0, x + 2y > 0 \)

or \( x < 0, x + 2y < 0. \)

Negative when \( x > 0, x + 2y < 0 \)

or \( x < 0, x + 2y > 0. \)

\([\text{Indefinite!}]\)
7. (a) $z = xy$. The domain of this function is the entire $xy$-plane.

(b) Trivial.

(c) Domain is $x \geq 0, y \in \mathbb{R}$, so the right half-plane.

(d) $z = x^2 - y^2 = 0$. Domain is all of $\mathbb{R}$, level set at $z = c$ is a circle of radius $\sqrt{c}$. Graph is a paraboloid.

(e) $z = x^2 + y^2$. Domain is all of $\mathbb{R}$, level set at $z = c$ is a circle of radius $c$. Graph is a cone.

(f) $xyz = 1$. Domain is set of points $(x, y)$ such that $x \neq 0$ and $y \neq 0$, so the $xy$ plane w/ coordinate axes removed. Level sets are hyperbolas.

(g) $\frac{xy}{z^2} = 1$. Here we can't have $xy = 0$, otherwise this has no solution. Also can't have $xy$ be negative, so the domain is the first and third quadrants of $xy$ plane. Level sets are hyperbolas.
11. (c) \( f(x, y) = \sqrt[3]{x \cdot y} \).

This function only makes sense when \( x \geq 0 \) and \( y \geq 0 \), so this is defined on the closed first quadrant in the xy-plane.

(d) \( f(x, y) = \sqrt{xy} \).

This function only makes sense when \( xy \geq 0 \), so this is defined when \( x \geq 0 \) and \( y \geq 0 \) or when \( x \leq 0 \) and \( y \leq 0 \), so the domain is the closed first and third quadrants in the xy-plane.

14. (a) \( f(x, t) = xsin(t) \).

So, as time progresses, this line rocks back and forth between a slope of 1 and -1.

(b) Almost same as (a), but this time the line is rocking twice as fast.

(c) \( f(x, t) = t \cdot sin(x) \).

As time progresses, we have a sine curve whose amplitude increases.
(f) \( f(x,t) = (x-t)^2 \).

As time progresses, we have a parabola that moves to the right.

(g) \( f(x,t) = (x-\sin t)^2 \).

As time progresses, we have a parabola that oscillates from right to left.