1. Trivial. Solutions are in answer key.

2. The answer is a function of \( x \), not a number. See explanation in answer key.

3. See explanation in answer key.

4. The volume of the rectangular block is \( ab \left( a^2 + b^2 \right) \) since the base of the block has area \( ab \) and the height of the block is \( a^2 + b^2 \). The volume of the portion lying under the graph of \( z = x^2 + y^2 \) is \( \int_0^b \int_0^a x^2 + y^2 \, dx \, dy = \frac{1}{3} ab \left( a^2 + b^2 \right) \). Therefore the fraction of the volume of the block under the graph of \( z = x^2 + y^2 \) is

\[
\frac{\frac{1}{3} ab \left( a^2 + b^2 \right)}{ab(a^2 + b^2)} = \frac{1}{3}.
\]

5. (a) Trivial. \( \int_0^1 \int_0^1 1 + x \, dx \, dy \)

(c) Trivial. \( \int_0^1 \int_0^y xy \, dx \, dy \).

(f) Domain is region under parabola.

\[
\int_0^1 \int_0^{\frac{x^2}{e^x}} y \, dy \, dx.
\]

Use integration by parts repeatedly.

\[
= \int_0^1 \frac{x^4}{2e^x} \, dx.
\]

(h) Domain is same as in (f).

\[
= \int_0^1 \int_0^{\sqrt{x^3 + 1}} dx \, dy.
\]

Swap order of integration to make the integral easier.

\[
= \int_0^1 \int_0^{x^2} dy \, dx = \int_0^1 x^{2/3} \, dx.
\]

Use substitution to solve.
6. (a) \( z = x^2 + y^2 \) is a paraboloid, and \( z = 4 \) is a plane. The region they bound is a solid "bowl."

This region in space lies over the disc of radius 2 in the \( xy \)-plane.

\[ D = \{(x,y) : x^2 + y^2 \leq 4 \} \]

\( = \{(x,y) : -2 \leq x \leq 2, -\sqrt{4-x^2} \leq y \leq \sqrt{4-x^2} \} \)

Polar coordinates \( = \{(r,\theta) : 0 \leq r \leq 2, 0 \leq \theta \leq 2\pi \} \)

The volume of the region is the volume under the plane but over the paraboloid.

So the volume is given by subtracting off the volume under the paraboloid from the volume under the plane.

\[ V = \iint_D (4 - (x^2 + y^2)) \, dA \]

(in rectangular coordinates) \( = \int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} (4 - (x^2 + y^2)) \, dy \, dx \)

(in polar coordinates) \( = \int_{0}^{2\pi} \int_{0}^{2} (4 - r^2) \, r \, dr \, d\theta \)

\[ = 8\pi \]
6 (d) Region is in the first octant \((x, y, z > 0)\) and is bounded by \(x+y+z=9\), \(2x+3y=18\), \(x+3y=9\).

This is the region trapped between these three planes. It is bounded above by the graph of \(x+y+z=9\) and lies over the region in the \(xy\)-plane bounded by the \(x\)-axis, \(y\)-axis and the lines \(2x+3y=18\) and \(x+3y=9\).

\[
V = \int_0^9 \int_{\frac{x}{2}}^{\frac{9-x}{3}} (9-x-y) \, dy \, dx = \frac{81}{2}.
\]

(e) Region is in the first octant and bounded by \(z^2 = a^2\), \(z = x+y\).

\(z^2 = a^2\) is a cylinder of radius \(a\).

\(z = x+y\) is a plane.

The region is bounded above by the plane and lies over the quarter-circle \(D\) in the \(xy\)-plane.

\[
D = \{(x, y) : x \geq 0, y \geq 0, x^2 + y^2 \leq a^2\} = \{(x, y) : 0 \leq x \leq a, 0 \leq y \leq \sqrt{a^2 - x^2}\}.
\]

\[
D = \{(r, \theta) : 0 \leq r \leq a, 0 \leq \theta \leq \frac{\pi}{4}\}.
\]

\[
V = \int_0^a \int_0^{\sqrt{a^2-x^2}} (x+y) \, dy \, dx = \int_0^{\frac{\pi}{4}} \int_0^a r (\cos \theta + \sin \theta) \, r \, dr \, d\theta = \frac{2a^3}{3}.
\]
7. \[
\text{Average} = \frac{\iiint_D e^{-\sqrt{x+y}} \, dA}{V} = \frac{1}{V} \int_0^V \int_0^1 \int_0^y e^{-\sqrt{x+y}} \, dy \, dx \quad \text{use substitution} \quad u = e^y, \quad du = e^y \, dy.
\]

\[
= \frac{1}{V} \int_0^y \int_0^1 e^{-\sqrt{x+u}} \, du \, dx
\]

\[
= \frac{1}{V} \int_0^y \left[ \frac{2}{3} (x+u)^{3/2} \right]_{u=0}^{u=1} \, dx
\]

\[
= \frac{1}{V} \int_0^y \left[ \frac{2}{3} (x+1)^{3/2} - \frac{2}{3} (x+1)^{3/2} \right] \, dx
\]

\[
= \frac{1}{b} \int_0^y (x+e)^{3/2} - (x+1)^{3/2} \, dx
\]

\[
= \frac{1}{b} \left[ \frac{2}{5} (x+e)^{5/2} - (x+1)^{5/2} \right]_0^{y=0}
\]

\[
= \frac{1}{15} \left( (4+e)^{5/2} - (5)^{5/2} - e^{5/2} + 1 \right).
\]

8. (a) \[
I = \int_0^b \int_0^b dy \, dx. \quad \text{Volume underneath the plane } z=1 \text{ over the region } D.
\]

\[
D = \{ (x,y) : 0 \leq x \leq b, \quad 0 \leq y = (b-x)^3 \} \text{ in the xy-plane}.
\]

But the volume under this plane is just the area of the region D since the height of the region is just the constant 1. Thus

\[
I = \text{area of } D = A, \quad \text{by definition of } A.
\]