Math 234
QUIZ 3

Name:

1. Classify the following quadratic forms as definite, indefinite, or semidefinite by completing the square. Sketch the region in the plane where the function is positive.

(a) \( f(x, y) = x^2 + 6xy + 11y^2. \)
\[
= (x^2 + 6xy + 9y^2) + 2y^2 \\
= (x + 3y)^2 + 2y^2
\]

Since this is a sum of squares, this is a definite quadratic form.
It will be positive everywhere except the origin.

(b) \( g(x, y) = 2x^2 + xy - y^2. \)
\[
= (2x - y)(x + y)
\]

Since this is a product of two linear terms, this is an indefinite quadratic form.

Zero when \( 2x - y = 0 \)
on \( x + y = 0 \)
2. Consider the equation $4z - x^2 - 4y^2 = 0$.

(a) What function $z = f(x, y)$ is defined by this equation? What is the largest domain of definition?

$$z = f(x, y) = \frac{x}{4} + y^2$$

The domain of definition is every point on the $xy$ plane: $(x, y) \in \mathbb{R}^2$.

Note that $\mathbb{R}$ or all real numbers is incorrect since the input for the function must be a pair of numbers, not a single number.

(b) Sketch the level sets of this function for $z = 0, z = 1, z = 4, z = 9$.

3. **Bonus**: Where's the best place to get a cup of coffee in Madison?

   My favorite place is Bradbury's, on the north corner of the capitol square. They also have excellent crepes there.