Math 234
QUIZ 9

Name:

1. Find the average value of the function \( f(x, y) = \cos x \cos y \) along the line segment from \((0,0)\) to \((\pi, \pi)\).

Parametrize the line as \( \vec{X}(t) = (t, t) \), \( 0 \leq t \leq \pi \).

We then have \( f(\vec{X}(t)) = f(t, t) = \cos t \cos t = \cos^2 t \).

\[ \vec{X}'(t) = (1, 1) \quad \| \vec{X}'(t) \| = \sqrt{2}. \]

\[
\int f \, ds = \int_0^\pi \cos^2 t \sqrt{2} \, dt = \sqrt{2} \int_0^\pi \cos^2 t \, dt = \sqrt{2} \int_0^\pi \frac{1 + \cos(2t)}{2} \, dt
\]

\[
= \sqrt{2} \left[ \frac{1}{2} t + \frac{\sin(2t)}{4} \right]_0^\pi
\]

\[
= \sqrt{2} \frac{\pi}{2}.
\]

Average value is \( \frac{1}{L} \int f \, ds \), where \( L \) is the length of the line segment.

\( L = \sqrt{2} \pi \), so average is \( \frac{1}{\sqrt{2} \pi} \cdot \frac{\pi - 0}{2} = \frac{1}{2} \).
2. Let $\vec{F} = (y, x)$. Compute the line integral $\int_C \vec{F} \cdot d\vec{x}$, where the curve $C$ is the ellipse \[ \frac{x^2}{4} + \frac{y^2}{9} = 1, \] oriented counterclockwise.

Note that $\vec{F}$ is a conservative vector field since $\vec{F} = \nabla f$, where $f = xy$.

Since $\vec{F}$ is conservative and $C$ is a closed curve, $\int_C \vec{F} \cdot d\vec{x} = 0$.

If you didn't see that $\vec{F}$ is conservative, you can show directly that the integral is zero.

Parameterize $C$ by $\vec{r}(t) = (2\cos t, 3\sin t)$, $0 \leq t \leq \pi$.

Then $\vec{r}'(t) = (-2\sin t, 3\cos t)$.

Thus $\int_C \vec{F} \cdot d\vec{x} = \int_0^{2\pi} (2\sin t, 3\cos t) \cdot (-2\sin t, 3\cos t) \, dt = \int_0^{2\pi} 4\sin^2 t + 9\cos^2 t \, dt$

$= 6 \int_0^{2\pi} \cos^2 t - \sin^2 t \, dt$ using $\cos 2t = \cos^2 t - \sin^2 t$.

$= 6 \left[ \frac{\sin(2t)}{2} \right]_0^{2\pi}$

$= 0$.

3. **Bonus**: Pick one of the following and explain why you made your choice:

(a) Flim
(b) Rhubarb
(c) Fingerbib
(d) Jynwceyhek

These are all pretty good. Check them out.