234 Worksheet 14

Topics: Triple integrals, integration in cylindrical and spherical coordinates

Exercises:

1. Let $D$ be a spherical igloo contained in the region between $x^2 + y^2 + z^2 = 16$ and $z \geq 0$. Suppose that the temperature $T$ of a point inside the igloo is given by $T(x, y, z) = 32 - 2(x^2 + y^2 + z^2)$. What is the average temperature inside the igloo?

2. A solid region the the first octant is bounded by the coordinate planes and the plane $x + y + z = 2$. The density of this solid is given by $2x$. Find the mass of the solid.

3. Consider the integral $\int_{-1}^{1} \int_{0}^{\sqrt{1-y^2}} \int_{0}^{\sqrt{x^2+y^2}} xyz \, dz \, dx \, dy$. Convert this integral to an integral in cylindrical coordinates.

4. Consider the integral $\int_{0}^{3} \int_{0}^{\sqrt{9-y^2}} \int_{0}^{\sqrt{18-x^2-y^2}} \sqrt{x^2+y^2} \, x^2 + y^2 + z^2 \, dx \, dy \, dz$. Convert this to an integral in spherical coordinates.