234 Worksheet 7

Topics: Clairaut’s theorem, Finding a function from its derivatives, Maxima and minima

Exercises:

1. Find all the second order derivatives for \( f(x, y) = \cos(2x) - x^2e^{5y} + 3y^2 \).

2. If \( \frac{\partial f}{\partial x} = Ax^2 + 2xy + y \) and \( \frac{\partial f}{\partial y} = Bx^2 + Cy + x \), then what values can \( A, B, C \) take?

3. Given the following functions \( P \) and \( Q \) find a function \( f \) such that \( \frac{\partial f}{\partial x} = P \) and \( \frac{\partial f}{\partial y} = Q \) or show that no such function exists.
   (a) \( P(x, y) = e^{xy} + 5y, \ Q(x, y) = e^{xy} + 5x \).
   (b) \( P(x, y) = \cos(xy^2)y^2 + e^{xy}2xy, \ Q(x, y) = \cos(xy^2)2xy + e^{xy}x^2 + 3 \).

4. For each of the following functions, find all the critical points.
   (a) \( f(x, y) = 10 + x^3 + y^3 - 3xy \).
   (b) \( f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4 \).

5. Consider the function \( f(x, y) = (y - x^2)(y - 2x) = y^2 - 2xy - x^2y + 2x^3 \).
   (a) Draw the zero set of \( f \) and determine the sign of each region cut out by the zero set.
   (b) Find the critical points of the function.
   (c) Without using the second derivative test, for each critical point, determine if the point is a max, a min, or a saddle point.