

## Math 222 Exam 1 Review Problems

**Problem 1** Compute the following integrals

(a)  $\int x^4 \cos x \, dx$

(b)  $\int \tan^3 \theta \, d\theta$

(c)  $\int \frac{dt}{(t+1)\sqrt{t^2+2t}}$

(d)  $\int \frac{dy}{y^{7/6}+y}$

(e)  $\int \cos^4 x \, dx$

(f)  $\int x \sec^2 x \, dx$

**Problem 2** Evaluate the following improper integrals

(a)  $\int_0^\infty \frac{dx}{x^2+2x+1}$

(b)  $\int_1^2 \frac{x \, dx}{(x^2-1)^{1/2}}$

(c)  $\int_0^\infty e^{-t} \sin t \, dt$

**Problem 3** Determine whether the following integrals converge or diverge.

(a)  $\int_1^\infty \frac{dx}{x-\sin^2 x}$

(b)  $\int_0^{1/2} \frac{dx}{x \ln x}$

(c)  $\int_1^\infty \frac{\cos^2 x \, dx}{e^{x^2}+1}$

**Problem 4** Solve the following initial value problem for  $x$  as a function of  $t$ .

$$(t^2 + 2t + 1) \frac{dx}{dt} = t^3 + 2 \text{ where } t > -1, x(0) = 2$$

**Problem 5** Find Cartesian equations for the following.

(a)  $r \cos \theta = r^2 + \tan \theta$

(b)  $r^2 \sin(2\theta) + \frac{1}{r} \sec \theta = 1$

**Problem 6** An ellipse with a focus at the origin has its semimajor axis of length 2 and an eccentricity of  $1/3$ . Find a polar equation for the ellipse, then find its directrix.

**Problem 7** Find the eccentricity of the following conic section. **Hint:** First find the center.

$$x^2 - y^2 - 4x + 4 = 1$$

**Problem 8** Rotate the coordinate axes to transform the given equation into an equation with no cross product ( $xy$ ) term.

$$2x^2 + \sqrt{3}xy + y^2 = 0$$

**Problem 9** Find the length of the curve  $r = \theta$  for  $0 \leq \theta \leq 1$ .

**Problem 10**

(a) Find the points of intersection of the curves  $r = 3 \sin \theta$  and  $r = 1 + \sin \theta$ . **Hint:** It may be helpful to sketch the two curves together.

(b) Find the area outside the curve  $r = 1 + \sin \theta$  and inside the curve  $r = 3 \sin \theta$ . **Hint:** It may be easier to perform the calculations if you use symmetry.