Problem 1 Compute the following integrals

(a) \( \int x^4 \cos x \, dx \)
(b) \( \int \tan^3 \theta \, d\theta \)
(c) \( \int \frac{dt}{(t+1)\sqrt{t^2 + 2t}} \)
(d) \( \int \frac{dx}{y\sqrt{x+y}} \)
(e) \( \int \cos^4 x \, dx \)
(f) \( \int x \sec^2 x \, dx \)

Problem 2 Evaluate the following improper integrals

(a) \( \int_0^\infty \frac{dx}{x^2 + 2x + 1} \)
(b) \( \int_1^{2/3} \frac{x \, dx}{(x^2 - 1)^{1/2}} \)
(c) \( \int_0^\infty e^{-t} \sin t \, dt \)

Problem 3 Determine whether the following integrals converge or diverge.

(a) \( \int_1^\infty \frac{dx}{x - \sin x} \)
(b) \( \int_0^{1/2} \frac{dx}{x \ln x} \)
(c) \( \int_1^\infty \frac{\cos^2 x \, dx}{e^{2x} + 1} \)

Problem 4 Solve the following initial value problem for \( x \) as a function of \( t \).

\( (t^2 + 2t + 1) \frac{dx}{dt} = t^3 + 2 \) where \( t > -1, x(0) = 2 \)

Problem 5 Find Cartesian equations for the following.

(a) \( r \cos \theta = r^2 + \tan \theta \)
(b) \( r^2 \sin(2\theta) + \frac{1}{2} \sec \theta = 1 \)

Problem 6 An ellipse with a focus at the origin has its semimajor axis of length 2 and an eccentricity of 1/3. Find a polar equation for the ellipse, then find its directrix.

Problem 7 Find the eccentricity of the following conic section. Hint: First find the center.

\( x^2 - y^2 - 4x + 4 = 1 \)

Problem 8 Rotate the coordinate axes to transform the given equation into an equation with no cross product \((xy)\) term.

\( 2x^2 + \sqrt{3}xy + y^2 = 0 \)

Problem 9 Find the length of the curve \( r = \theta \) for \( 0 \leq \theta \leq 1 \).

Problem 10

(a) Find the points of intersection of the curves \( r = 3\sin \theta \) and \( r = 1 + \sin \theta \). Hint: It may be helpful to sketch the two curves together.
(b) Find the area outside the curve \( r = 1 + \sin \theta \) and inside the curve \( r = 3\sin \theta \). Hint: It may be easier to perform the calculations if you use symmetry.