Math 222 Exam 2 Review Problems

Problem 1 Determine whether the following sequences $\{a_n\}$ converge or diverge. Find the limit of each of the convergent sequences.

(a) $a_n = \left(1 + \frac{3}{n}\right)^n$

(b) $a_n = (-1)^n + 3$

(c) $a_n = \frac{25^n}{n!}$

(d) $a_n = \frac{\sin n}{n^2}$

(e) $a_n = \sqrt{\frac{n^3+5n^2+4}{3n^3-6}}$

Problem 2 Determine whether the following series converge or diverge. Find the sum of each of the convergent series.

(a) $\sum_{n=0}^{\infty}(-1)^{n+1}\frac{3^n}{5^n}$

(b) $\sum_{n=0}^{\infty}\left(\frac{2}{\sqrt{2}}\right)^n$

(c) $\sum_{n=0}^{\infty}\frac{1}{2^{n+2}}$

Problem 3 Determine whether the following series converge or diverge.

(a) $\sum_{n=2}^{\infty}\frac{1}{n(\ln n)^3}$

(b) $\sum_{n=1}^{\infty}\frac{5}{n^{3/2}+4}$

(c) $\sum_{n=1}^{\infty}\frac{n^3}{n^4+n^3-1}$

(d) $\sum_{n=1}^{\infty}\frac{\ln n}{n^2}$

(e) $\sum_{n=1}^{\infty}\left(\frac{e^n}{n}\right)^n$

Problem 4 Determine whether the following series converge absolutely, converge conditionally, or diverge.

(a) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{2^n}{n!}$

(b) $\sum_{n=2}^{\infty}(-1)^{n-1}\frac{n}{\ln n}$

(c) $\sum_{n=1}^{\infty}(-1)^{n}\frac{\sqrt{n}}{2n-1}$

Problem 5 Determine where the series $\sum_{n=0}^{\infty}(e^x)^n$ converges and, within its interval of convergence, the sum of the series as a function of x.

Problem 6 Find the interval of convergence of each of the following power series

(a) $\sum_{n=0}^{\infty} n^n x^n$

(b) $\sum_{n=0}^{\infty}\frac{(x-1)^n}{\sqrt{n}}$

Problem 7 Find a power series that converges to $\frac{x}{(1-x)^2}$ on the interval $-1 < x < 1$.
Hint: $\frac{d}{dx}\left(\frac{1}{1-x}\right) = \frac{1}{(1-x)^2}$.

Problem 8 For each of the following, find the third order Taylor polynomial generated by f at a.

(a) $f(x) = x\sin x$, $a = \pi/2$

(b) $f(x) = (\ln x)^2$, $a = 1$

Problem 9 Find the Maclaurin series for $f(x) = xe^{-x^2}$.

Problem 10 Find the sum of the series $\sum_{n=0}^{\infty}\frac{(-1)^n(\pi/2)^{2n}}{(2n)!}$.

Problem 11 Find a bound on the error made by estimating $\cos x$ with $1 - \frac{x^2}{2}$ for $|x| < 0.2$