Problem 1 Determine whether the following sequences \( \{a_n\} \) converge or diverge. Find the limit of each of the convergent sequences.

(a) \( a_n = \left(1 + \frac{2}{n}\right)^n \)
(b) \( a_n = (-1)^n + 3 \)
(c) \( a_n = \frac{2^n}{n!} \)
(d) \( a_n = \frac{\sin n}{n^2} \)
(e) \( a_n = \sqrt{n^3 + 5n^2 + 4n^3 - 6} \)

Problem 2 Determine whether the following series converge or diverge. Find the sum of each of the convergent series.

(a) \( \sum_{n=0}^\infty (-1)^n n \cdot 3^n \)
(b) \( \sum_{n=0}^\infty \left(\frac{2}{\sqrt{n}}\right)^n \)
(c) \( \sum_{n=0}^\infty \frac{1}{2n+2} \)

Problem 3 Determine whether the following series converge or diverge.

(a) \( \sum_{n=2}^\infty \frac{1}{n(n+1)^2} \)
(b) \( \sum_{n=1}^\infty \frac{5}{n^2 + 4} \)
(c) \( \sum_{n=1}^\infty \frac{n^3}{n^2 + n - 1} \)
(d) \( \sum_{n=1}^\infty \ln n \)
(e) \( \sum_{n=1}^\infty \left(\frac{\pi}{n}\right)^n \)

Problem 4 Determine whether the following series converge absolutely, converge conditionally, or diverge.

(a) \( \sum_{n=1}^\infty (-1)^n + \frac{2^n}{n!} \)
(b) \( \sum_{n=2}^\infty (-1)^n - \frac{n}{\ln n} \)
(c) \( \sum_{n=1}^\infty (-1)^n \frac{\sqrt{n}}{2n-1} \)

Problem 5 Determine where the series \( \sum_{n=0}^\infty (e^x)^n \) converges and, within its interval of convergence, the sum of the series as a function of \( x \).

Problem 6 Find the interval of convergence of each of the following power series

(a) \( \sum_{n=0}^\infty n^n \cdot x^n \)
(b) \( \sum_{n=0}^\infty \frac{(x-1)^n}{\sqrt{n}} \)

Problem 7 Find a power series that converges to \( \frac{1}{1-x} \) on the interval \(-1 < x < 1\).

Hint: \( \frac{d}{dx} \left(\frac{1}{1-x}\right) = \frac{1}{(1-x)^2} \).

Problem 8 For each of the following, find the third order Taylor polynomial generated by \( f \) at \( a \).

(a) \( f(x) = x \sin x, \ a = \pi/2 \)
(b) \( f(x) = (\ln x)^2, \ a = 1 \)

Problem 9 Find the Maclaurin series for \( f(x) = xe^{-x^2} \).

Problem 10 Find the sum of the series \( \sum_{n=0}^\infty (-1)^n (\pi/2)^n (2n)! \).

Problem 11 Find a bound on the error made by estimating \( \cos x \) with \( 1 - \frac{x^2}{2} \) for \( |x| < 0.2 \).