

## Math 222 Exam 3 Review Problems

**Problem 1** Evaluate the following integrals

(a)  $\int e^{2x} \cos(2x) dx$

(b)  $\int \arcsin y dy$

(c)  $\int \frac{-x^3 + x^2 - 3x + 1}{(1-x)^2(1+x^2)} dx$

(d)  $\int_0^{3/2} \sqrt{9-4x^2} dx$

**Problem 2** Evaluate the following improper integral, or show that it diverges.

$$\int_e^\infty \frac{dt}{t(\ln t)^2}$$

**Problem 3** Find an equation for the ellipse with foci at  $(\pm 2, 0)$  and vertices at  $(\pm\sqrt{5}, 0)$ , then find its eccentricity.

**Problem 4** Find the center and eccentricity of the hyperbola given by  $2x^2 - y^2 - 2y = 3$ .

**Problem 5** Rotate the coordinate axes to eliminate the  $xy$  term in the conic section  $x^2 + xy + y^2 = 1$ . What type of conic section is it?

**Problem 6** Convert the polar equation  $r^2 \sin(2\theta) = 1$  to Cartesian coordinates.

**Problem 7** Find the area shared by the graphs of  $r_1 = 1 - \cos \theta$  and  $r_2 = 1 + \cos \theta$ .

**Problem 8** Find  $k$  and  $e$  for the conic section given by the equation  $r = \frac{3}{2 + 6 \cos \theta}$ .

**Problem 9** Find the sums of the following series

(a)  $\sum_{n=0}^{\infty} \frac{3^{n-1}}{2^{n+3}5^{n-2}}$

(b)  $\sum_{k=0}^{\infty} \left[ \frac{3^k}{k!} - \frac{(-1)^k (\pi/2)^{2k+1}}{(2k+1)!} \right]$

**Problem 10** Determine whether the following series converge or diverge

(a)  $\sum_{n=3}^{\infty} n \left( \frac{n+5}{2n-1} \right)^n$

(b)  $\sum_{n=0}^{\infty} \left[ 1 + \left( \frac{1}{2} \right)^n \right]$

(c)  $\sum_{n=5}^{\infty} \frac{3n+5}{n^2-1}$

(b)  $\sum_{k=2}^{\infty} \frac{(-1)^k}{\ln k}$

**Problem 11** Determine where the power series  $\sum_{n=1}^{\infty} \frac{2^n}{n} x^n$  converges absolutely and where it converges conditionally.

**Problem 12** Find the Maclaurin series for  $f(x) = \arctan x$ .

**Problem 13** Solve the differential equation  $\frac{dy}{dx} = \cos(x+y) + \sin x \sin y$ .

**Problem 14** Solve the following initial value problems.

(a)  $x^2 y' = y + x^3 e^{-(1/x)}$ ,  $x > 0$ ,  $y(1) = \frac{1}{e}$

(b)  $y'' - 6y' + 9y = 0$ ,  $y(0) = 1$ ,  $y'(0) = 5$

**Problem 15** Solve the following differential equations.

(a)  $y'' + y' - 2y = e^x$

(b)  $y'' + y' + 2y = x^2$

**Problem 16**

(a) Use dot products to show that the vectors  $\mathbf{u} + \mathbf{v}$  and  $\mathbf{u} - \mathbf{v}$  are orthogonal if and only if  $|\mathbf{u}| = |\mathbf{v}|$ .

(b) Suppose that  $AB$  is the diameter of a circle with center  $O$  and  $C$  is a point on one of the two arcs joining  $A$  and  $B$ . Use dot products to show that  $\overrightarrow{CA}$  and  $\overrightarrow{CB}$  are orthogonal.

**Problem 17** Let  $\mathbf{u} = \mathbf{j} + 2\mathbf{k}$ ,  $\mathbf{v} = \mathbf{i} + \mathbf{j} + \mathbf{k}$  and  $\mathbf{w} = \mathbf{i} - \mathbf{j} + \mathbf{k}$ . Find the volume of the box determined by  $\mathbf{u}$ ,  $\mathbf{v}$  and  $\mathbf{w}$ .

**Problem 18** Let  $\mathbf{u} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}$  and let  $\mathbf{v} = \mathbf{j} - \mathbf{k}$ .

(a) Find the cosine of the angle between  $\mathbf{u}$  and  $\mathbf{v}$ .

(b) Write  $\mathbf{u}$  as the sum of a vector parallel to  $\mathbf{v}$  and a vector orthogonal to  $\mathbf{v}$ .

**Problem 19** Do there exist nonzero vectors  $\mathbf{u}$  and  $\mathbf{v}$  such that  $\mathbf{u} \cdot \mathbf{v} = 0$  and  $\mathbf{u} \times \mathbf{v} = \mathbf{0}$ ?

**Problem 20** Consider the points  $A(0, 1, 1)$ ,  $B(1, 2, 1)$  and  $C(3, 0, 0)$ .

(a) Find the area of the triangle  $ABC$ .

(b) Find the equation of the plane passing through  $A$ ,  $B$  and  $C$ .

(c) Find the distance from the point  $S(1, 1, 1)$  to the plane determined in part (b).

(d) Parametrize the line through  $S$  that is orthogonal to the plane determined in part (b).

**Problem 21** Consider the two planes  $2x + y + z = 2$  and  $x - y = 1$ .

(a) Find a parametrization for the line of intersection of the two planes.

(b) Find the distance from the point  $S(2, 1, 0)$  to the line determined in part (a).