Math 222 Exam 3 Review Solutions

Problem 1

(a) $\dfrac{1}{4}e^{2x}\left[\cos(2x)+\sin(2x)\right]+C$

(b) $y\arcsin y+\sqrt{1-y^2}+C$

(c) $\arctan x-\dfrac{1}{1-x}-\ln|1-x|+C$

(d) $\dfrac{9\pi}{8}$

Problem 2

1

Problem 3

$\frac{x^2}{5}+y^2=1$, $e=\frac{2}{\sqrt{5}}$

Problem 4

Center is $(0,1)$. Eccentricity is $\sqrt{3}$.

Problem 5

$\dfrac{3}{2}(x')^2+\dfrac{1}{2}(y')^2=1$. This represents an ellipse.

Problem 6

$2xy=1$

Problem 7

$\dfrac{3\pi}{2}-4$

Problem 8

$k=1/2$, $e=3$

Problem 9

(a) $\dfrac{250}{168}$

(b) e^3-1

Problem 10

(a) Converges; Root test

(b) Diverges; n^{th}-term test

(c) Diverges; Limit comparison test

(b) Converges; Alternating series test

Problem 11

Converges absolutely for $-\dfrac{1}{2}<|x|<\dfrac{1}{2}$. Converges conditionally for $x=-\dfrac{1}{2}$.

Problem 12

$$\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{2n+1}$$

Problem 13

$\ln|\sec y + \tan y| = \sin x + C$

Problem 14

(a) $y = \frac{1}{2}x^2 e^{-(1/x)} + \frac{1}{2}e^{-(1/x)}$

(b) $y = e^{3x} + 2xe^{3x}$

Problem 15

(a) $y = c_1 e^x + c_2 e^{-2x} + \frac{1}{3}xe^x$

(b) $y = e^{-1/2}\left[c_1 \cos\left(\frac{\sqrt{7}}{2}\right) + c_2 \sin\left(\frac{\sqrt{7}}{2}\right)\right] + \frac{1}{2}x^2 - \frac{1}{2}x - \frac{1}{4}$

Problem 16

(a) We find that $(\mathbf{u}+\mathbf{v})\cdot(\mathbf{u}-\mathbf{v}) = |\mathbf{u}|^2 - |\mathbf{v}|^2$, which is equal to zero if and only if $|\mathbf{u}| = |\mathbf{v}|$.

(b) If we let $\mathbf{u} = \overrightarrow{OB}$ and $\mathbf{v} = \overrightarrow{OC}$, then we can show that $\overrightarrow{CA} = -(\mathbf{u}+\mathbf{v})$ and that $\overrightarrow{CB} = (\mathbf{u}-\mathbf{v})$. Using the fact that $|\mathbf{u}| = |\mathbf{v}|$ (since both quantities will be equal to the radius of the circle) we find that $\overrightarrow{CA}\cdot\overrightarrow{CB} = 0$.

Problem 17

4

Problem 18

(a) $\frac{1}{\sqrt{12}}$

(b) $\mathbf{u} = \langle 0, 1/2, -1/2\rangle + \langle 1, 3/2, 3/2\rangle$

Problem 19

No. $\mathbf{u}\cdot\mathbf{v} = 0$ would imply that $\mathbf{u}$ and $\mathbf{v}$ are perpendicular, while $\mathbf{u}\times\mathbf{v} = \mathbf{0}$ would imply that $\mathbf{u}$ and $\mathbf{v}$ are parallel. Two nonzero vectors cannot be both parallel and perpendicular.

Problem 20

(a) $\sqrt{6}/2$

(b) $x - y - 2z = -3$

(c) $1/\sqrt{6}$

(d) One possible parametrization is $x = 1+t$, $y = 1-t$ and $z = 1-2t$.

Problem 21

(a) One possible parametrization is $x = 1+t$, $y = t$, $z = -3t$.

(b) $\sqrt{\frac{18}{11}}$