Math 234 Practice Problems for Exam 1

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Directions: Answer each of the following questions. Solutions are on the last page.

Problem 1. Find each of the following limits, or show that they don't exist.

(a) $\lim_{(x,y)\to(0,0)} \dfrac{xy^2}{x^2+y^4}$

(b) $\lim_{(x,y)\to(0,0)} \dfrac{xy^2}{(x^2+y^2)\cos(x^2+y^2)}$

(c) $\lim_{(x,y)\to(0,1)} \dfrac{x+y-1}{xy}$

Problem 2. Let $z = f(x,y) = \dfrac{1}{2}x^2y + \sin y$. Suppose that $x = x(t)$ and $y = y(t)$ are differentiable functions of t. Suppose further that we know the following:

(i) When $t = 0$ we have that $x = 2$ and $y = \pi$

(ii) $\left.\dfrac{dx}{dt}\right|_{t=0} = \left.\dfrac{dy}{dt}\right|_{t=0}$

(iii) $\left.\dfrac{dz}{dt}\right|_{t=0} = e$

Find the derivatives $\dfrac{dx}{dt}$ and $\dfrac{dy}{dt}$ at $t = 0$.

Problem 3. Find all directions $\mathbf{u}$ such that the directional derivative of $f(x, y) = x^2 + y$ at the point $P(2, 5)$ is equal to 1.

Problem 4. Suppose the equation $xz + 3x^2y - e^{yz} = 0$ defines z as a differentiable function of x and y. Let $P_0 = (1, 0, 1)$.

(a) Find the direction $\mathbf{u}$ in which z increases most rapidly at P_0. What is the derivative of z in that direction?

(b) Find the direction $\mathbf{u}$ in which z decreases most rapidly at P_0. What is the derivative of z in that direction?

(c) Find all directions $\mathbf{u}$ in which the directional derivative of z at P_0 in direction $\mathbf{u}$ is equal to zero.

Problem 5. Suppose that $f(x, y)$ is a differentiable function such that the directional derivative of f at $P_0(1, 2)$ in the direction pointing towards $P_1(3, 4)$ is $\sqrt{2}$ and the directional derivative of f at P_0 in the direction pointing towards $P_2(2, 0)$ is $\sqrt{5}$.

(a) Is there a direction $\mathbf{u}$ in which the rate of change of f in direction $\mathbf{u}$ at P_0 is equal to 4?

(b) Is there a direction $\mathbf{v}$ in which the rate of change of f in direction $\mathbf{v}$ at P_0 is equal to -4?

Solutions

1a. Does not exist.

1b. 0

1c. Does not exist.

2. $\left.\dfrac{dx}{dt}\right|_{t=0} = \left.\dfrac{dy}{dt}\right|_{t=0} = \dfrac{e}{2\pi + 1}$

3. $\mathbf{j}$ and $\frac{8}{17}\mathbf{i} - \frac{15}{17}\mathbf{j}$

4a. $\mathbf{u} = -\dfrac{1}{\sqrt{5}}\mathbf{i} - \dfrac{2}{\sqrt{5}}\mathbf{j}$. $(D_{\mathbf{u}}f)_{P_0} = \sqrt{5}$

4b. $\mathbf{u} = \dfrac{1}{\sqrt{5}}\mathbf{i} + \dfrac{2}{\sqrt{5}}\mathbf{j}$. $(D_{\mathbf{u}}f)_{P_0} = -\sqrt{5}$

4c. $-\dfrac{1}{\sqrt{5}}\mathbf{i} + \dfrac{2}{\sqrt{5}}\mathbf{j}$ and $\dfrac{1}{\sqrt{5}}\mathbf{i} - \dfrac{2}{\sqrt{5}}\mathbf{j}$

5a. No. The rate of fastest increase of f at P_0 is $\sqrt{10}$, which is less than 4.

5b. No. The rate of fastest decrease of f at P_0 is $-\sqrt{10}$, which is greater than -4.