

# Math 234 Suggested Problems on Gradients and Directional Derivatives

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**Directions:** Answer each of the following questions. Solutions are on the second page.

**Problem 1.** Find all points  $P$  on the line  $y = \frac{x}{2}$  such that the directional derivative of  $f(x, y) = x^2y$  in the direction pointing towards  $Q(1, 0)$  is zero.

**Problem 2a.** Let  $C$  be the circle in the  $xy$ -plane with center  $(1, 0)$  and radius 1. Let  $f(x, y) = \frac{1}{2}x^2 - \frac{1}{3}y^3 + y$ . Find the point  $P_0$  on  $C$  and direction  $\mathbf{u}$  such that the directional derivative  $(D_{\mathbf{u}}f)_{P_0}$  is maximized.

(Recall that the circle  $C$  can be described by the equation  $(x - 1)^2 + y^2 = 1$ )

**Problem 2b.** For the point  $P_0$  and direction  $\mathbf{u}$  found in part (a), what is the value of  $(D_{\mathbf{u}}f)_{P_0}$ ?

## Solutions

1.  $(0, 0)$  and  $(\frac{2}{3}, \frac{1}{3})$ .

2a.  $P_0 = (2, 0)$ .  $\mathbf{u} = \frac{2}{\sqrt{5}}\mathbf{i} + \frac{1}{\sqrt{5}}\mathbf{j}$ .

2b.  $\sqrt{5}$ .