

## Math 234 Suggested Problems on Multiple Integrals

**Directions:** Solve each of the following problems. Solutions are on the second page.

1. Find the integral of  $f(x, y) = x^2 + y^2$  over the region  $R$  that lies inside the circle  $x^2 + y^2 = 2$  and outside the square  $0 \leq x \leq 1, 0 \leq y \leq 1$ .

2. Convert the integral  $\int_{\theta=0}^{\pi/4} \int_{r=0}^{\frac{\sin \theta}{\cos^2 \theta}} r \, dr \, d\theta$  to Cartesian coordinates. You do not need to evaluate the integral.

3. Convert the integral  $\int_{x=-2}^2 \int_{y=-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{z=0}^{\sqrt{4-x^2-y^2}} dz \, dy \, dx$  to spherical coordinates. You do not need to evaluate the integral.

4. Convert the integral  $\int_{\theta=\pi}^{2\pi} \int_{\phi=0}^{\pi/4} \int_{\rho=0}^2 \rho^3 \cos \phi \sin \phi \, d\rho \, d\phi \, d\theta$  to cylindrical coordinates. You do not need to evaluate the integral.

5. Convert the integral  $\int_{x=-1}^1 \int_{y=0}^{\sqrt{1-x^2}} \int_{z=-\sqrt{1-x^2-y^2}}^{1-\sqrt{x^2+y^2}} dz \, dy \, dx$  to cylindrical coordinates. You do not need to evaluate the integral.

## Solutions

1.  $2\pi - \frac{2}{3}$

2.  $\int_{x=0}^1 \int_{y=x^2}^x dy dx$

3.  $\int_{\theta=0}^{2\pi} \int_{\phi=0}^{\pi/2} \int_{\rho=0}^2 \rho^2 \sin \phi d\rho d\phi d\theta.$

4.  $\int_{\theta=\pi}^{2\pi} \int_{r=0}^2 \int_{z=0}^{\sqrt{4-r^2}} rz dz dr d\theta.$

5.  $\int_{\theta=0}^{\pi} \int_{r=0}^1 \int_{z=-\sqrt{1-r^2}}^{1-r} r dz dr d\theta.$