1 Problems from the book

Section 1.1: 15, 18, 31

Section 1.2: 2, 7(a)(f)

Section 1.3: 13(a), 15(c)

Section 1.4: 27

Section 1.6: 17

Section 2.1: 1(a), 3(a)

Section 2.2: 5, 7(c), 21

Section 2.3: 9(a)(c), 13

2 Definitions

1. What does it mean for an equation $E[u] = g$ to be linear?

2. What does it mean for a system of linear equations to be consistent? Inconsistent?

3. What does it mean for a system of linear equations
\begin{align*}
a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &= b_1 \\
a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &= b_2 \\
&\vdots \\
a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n &= b_m
\end{align*}

to be homogeneous? Inhomogeneous?

4. What does it mean for a solution to a homogeneous system of linear equations to be non-trivial?

5. What does it mean for an \( n \times n \) matrix \( A \) to be invertible?

6. What are the three elementary row operations?

### 3 Additional Problem

Recall that an \( n \times n \) matrix \( A \) is said to be *diagonal* if \( a_{ij} = 0 \) whenever \( i \neq j \) (i.e. if every entry that is not on the main diagonal is zero).

Show that a \( 2 \times 2 \) matrix \( A \) has the property that for all \( 2 \times 2 \) matrices \( B \), \( AB = BA \) if and only if \( A \) is diagonal and \( a_{11} = a_{22} \).

(Put another way, we need to show that a \( 2 \times 2 \) matrix \( A \) has the property that for all \( 2 \times 2 \) matrices \( B \), \( AB = BA \) if and only if \( A \) is a scalar multiple of \( I_2 \), the \( 2 \times 2 \) identity matrix).

**Hint:** To show the "and only if" part, it may be useful to consider the matrices

\[
B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}
\]