# Math 340 Review Problems for the Exam 1

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September 28, 2007

## 1 Problems from the book

Section 1.1: 15, 18, 31

Section 1.2: 2, 7(a)(f)

Section 1.3: 13(a), 15(c)

Section 1.4: 27

Section 1.6: 17

Section 2.1: 1(a), 3(a)

Section 2.2: 5, 7(c), 21

Section 2.3: 9(a)(c), 13

## 2 Definitions

**1.** What does it mean for an equation $E[u] = g$ to be linear?

**2.** What does it mean for a system of linear equations to be consistent? Inconsistent?

**3.** What does it mean for a system of linear equations

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &= b_1 \\
a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &= b_2 \\
&\vdots \\
a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n &= b_m
\end{aligned}
$$

to be homogeneous? Inhomogeneous?

**4.** What does it mean for a solution to a homogeneous system of linear equations to be non-trivial?

**5.** What does it mean for an $n \times n$ matrix $A$ to be invertible?

**6.** What are the three elementary row operations?

# 3 Additional Problem

Recall that an $n \times n$ matrix $A$ is said to be *diagonal* if $a_{ij} = 0$ whenever $i \neq j$ (i.e. if every entry that is not on the main diagonal is zero).

Show that a $2 \times 2$ matrix $A$ has the property that for *all* $2 \times 2$ matrices $B$, $AB = BA$ if and only if $A$ is diagonal *and* $a_{11} = a_{22}$.

(Put another way, we need to show that a $2 \times 2$ matrix $A$ has the property that for *all* $2 \times 2$ matrices $B$, $AB = BA$ if and only if $A$ is a scalar multiple of $I_2$, the $2 \times 2$ identity matrix).

**Hint:** To show the "and only if" part, it may be useful to consider the matrices

$$B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$