

Math 222 Review Problems for Exam 1

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Directions: Answer each of the following questions.

Question 1. Solve each of the following integrals.

(a) $\int x^4 e^x dx$

(b) $\int \frac{x dx}{(x^2 + 1)^3}$

(c) $\int \tan^3 x dx$

(d) $\int \tan^{-1} x dx$. (Recall that $\tan^{-1} x$ is the inverse tangent function, *not* tangent to the negative first power)

(e) $\int \frac{2x^2 - 4x + 1}{(x - 1)^2(x + 2)} dx$

(f) $\int x^2 \ln x dx$

(g) $\int \frac{x^2 dx}{\sqrt{x^2 - 1}}$

(h) $\int e^{2x} \cos x dx$

(i) $\int \sin^2(2x) \cos x \, dx$

(j) $\int \frac{x^4 + 1}{x^3 + x} \, dx$

(k) $\int \sqrt{16 - 4x^2} \, dx$

Question 2.

(a) Find the area of the surface obtained by rotating the portion of the graph of $y = \sin x$ between $x = 0$ and $x = \pi$ around the x -axis.

(b) Find the volume of the solid obtained by rotating the portion of the xy -plane bounded by the x -axis and the portion of the graph of $y = \sin^2 x$ between $x = 0$ and $x = \pi$ about the x -axis.

(c) Find the center of gravity of the region in the xy -plane bounded by the x -axis and the portion of the graph of $y = \sin^2 x$ between $x = 0$ and $x = \pi$. (**Hint:** You can use symmetry to determine the x -coordinate of the center of gravity).

Question 3. Consider the integral $\int_1^2 \frac{2 \, dx}{x}$

(a) Estimate the integral using the trapezoidal rule, with $n = 4$. Without computing the integral, give an upper bound for the error in your estimate.

(b) Estimate the integral using Simpson's rule, with $n = 4$. Without computing the integral, give an upper bound for the error in your estimate.

Question 4. Compute the following improper integrals.

(a) $\int_0^{\infty} x e^{-x} \, dx$

(b) $\int_0^1 \frac{e^x \, dx}{\sqrt{e^x - 1}}$

Question 5. Determine whether the following improper integrals converge or diverge.

$$(a) \int_1^{\infty} \frac{dx}{x^2 + \sqrt{x}}$$

$$(b) \int_1^2 \frac{dx}{x(\ln x)^{1/3}}$$

$$(c) \int_e^{\infty} \frac{x^2 dx}{x^3 - e^{1/x}}$$

Question 6. Find the foci, vertices, directrices and eccentricity of the conic section given by $4x^2 + 9y^2 = 36$. Sketch a graph of the conic section.

Question 7. Find the eccentricity, vertices and asymptotes of the conic section centered at the origin with a focus at $(2, 0)$ and associated directrix of $x = 1$. Sketch a graph of the conic section.

Question 8. Find the eccentricity, vertex, focus and directrix of the conic section given by $x^2 = 4y$. Sketch a graph of the conic section.