Math 222 Review Problems for Exam 1

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Directions: Answer each of the following questions.

Question 1. Solve each of the following integrals.

(a) \[ \int x^4 e^x \, dx \]

(b) \[ \int \frac{x \, dx}{(x^2 + 1)^3} \]

(c) \[ \int \tan^3 x \, dx \]

(d) \[ \int \tan^{-1} x \, dx. \) (Recall that \( \tan^{-1} x \) is the inverse tangent function, \textit{not} tangent to the negative first power)

(e) \[ \int \frac{2x^2 - 4x + 1}{(x - 1)^2(x + 2)} \, dx \]

(f) \[ \int x^2 \ln x \, dx \]

(g) \[ \int \frac{x^2 \, dx}{\sqrt{x^2 - 1}} \]

(h) \[ \int e^{2x} \cos x \, dx \]
(i) $\int \sin^2(2x) \cos x \, dx$

(j) $\int \frac{x^4 + 1}{x^3 + x} \, dx$

(k) $\int \sqrt{16 - 4x^2} \, dx$

**Question 2.**

(a) Find the area of the surface obtained by rotating the portion of the graph of $y = \sin x$ between $x = 0$ and $x = \pi$ around the $x$-axis.

(b) Find the volume of the solid obtained by rotating the portion of the $xy$-plane bounded by the $x$-axis and the portion of the graph of $y = \sin^2 x$ between $x = 0$ and $x = \pi$ about the $x$-axis.

(c) Find the center of gravity of the region in the $xy$-plane bounded by the $x$-axis and the portion of the graph of $y = \sin^2 x$ between $x = 0$ and $x = \pi$. 

(Hint: You can use symmetry to determine the $x$-coordinate of the center of gravity).

**Question 3.** Consider the integral $\int_1^2 \frac{2 \, dx}{x}$

(a) Estimate the integral using the trapezoidal rule, with $n = 4$. Without computing the integral, give an upper bound for the error in your estimate.

(b) Estimate the integral using Simpson’s rule, with $n = 4$. Without computing the integral, give an upper bound for the error in your estimate.

**Question 4.** Compute the following improper integrals.

(a) $\int_0^\infty x e^{-x} \, dx$

(b) $\int_0^1 \frac{e^x \, dx}{\sqrt{e^x - 1}}$

**Question 5.** Determine whether the following improper integrals converge or diverge.
(a) \[ \int_1^\infty \frac{dx}{x^2 + \sqrt{x}} \]

(b) \[ \int_1^2 \frac{dx}{x(\ln x)^{1/3}} \]

(c) \[ \int_e^\infty \frac{x^2 \, dx}{x^3 - e^{1/x}} \]

**Question 6.** Find the foci, vertices, directices and eccentricity of the conic section given by \(4x^2 + 9y^2 = 36\). Sketch a graph of the conic section.

**Question 7.** Find the eccentricity, vertices and asymptotes of the conic section centered at the origin with a focus at \((2, 0)\) and associated directrix of \(x = 1\). Sketch a graph of the conic section.

**Question 8.** Find the eccentricity, vertex, focus and directrix of the conic section given by \(x^2 = 4y\). Sketch a graph of the conic section.